What is a Derivative and What do we do with it?

What does the function represent? Functions represent the observance of almost anything from the rate at which something burns to how fast a school building empties at the end of the day. Many functions are the result of the physical laws of nature, others are the result of data collection like profit/loss functions where factors such as cost/market/weather/labor may be in effect, population growth functions of a species or bacteria depend on things like rate of reproduction, food, temp, etc.

Now that I have a function, what do I do with it? Most functions are affected by time or some factor 'x', so often we want to know what a function is doing at a particular point in time, or over a period of time. There are many tools we use to analyze a function, one in particular is the Derivative.

Methods to find a derivative: There are many ways to find a derivative, but they all result in an analysis of a function… a quick look at what is happening to the function at a specific point along the function.

What do we do with the Derivative?

If we observe a person throwing a ball up in the air and catching it, we would most likely observe it initially going up with some velocity, slowing to a stop and coming back down with increasing velocity. Through observation we can define its height in terms of a function ($h= -at^2 + v_0t + h_0$). To find the velocity (v) at any point in time, we would take the derivative of h(t).

1st Derivative: gives us the slope of the tangent line at any point along the f(x) and slope is equivalent to rate of change.

What do we do with slope?

Slope of the tangent gives us the **rate of change** of a function at some point.

Zero Slope: a zero slope indicates a local maximum/minimum point of the function. It is often considered a critical point in the function.

The Difference Quotient: the "long way" to find the derivative.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$ (Δx is sometimes represented by h)

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$ to quickly find the Dx at a specific value of x

The Diff Quotient is an algebraic approach to finding the derivative of an equation. The use of x + Δx where Δx is decreasing to zero, is the idea that the slope of a secant line becomes the slope of a tangent line at x as Δx goes to zero (Lim as $\Delta x \to 0$)

Secant line: Touches a curve at two points

Tangent line: Touches a curve at one point

DERIVATIVE The derivative is a limit. So for the derivative to exist, the limit must exist, and the function must be continuous at that point.

Exceptions: The Dx does not exist at a "corner", or "cusp" even if the function is continuous. Dx^{+} must equal Dx^{-} (similar to the idea of a Limit)

RESULTS: 1[ST] Der: Slope of the Tangent line at any point on the graph of an equation.
2[nd] Der: Concavity of an equation at any point.

CRITICAL POINTS (CP): Are determined by setting F'(x) or F''(x) equal to zero.

1[st] Der: Yields a point(s) at which slope for F(x) is equal to zero

- Local maxima/minima

2[nd] Der: Yields **Point of Inflection** for F(x) where concavity changes direction or where the slope changes from increasing to decreasing, or vice versa.

Don't Exist:

- 1[st] Der: Indicates no local maxima/minima, constant slope, or an undefined slope
- 2[nd] Der: Indicates no Inflection Point, no concavity
- Point of discontinuity

Symbols: Dx, f '(x), dy/dx, f ', f '', f^1, f^2

Facts:

- The Dx of a constant = 0 (a constant has no 'variable' component and thus no slope)
- You always take a derivative with respect to some variable (Dx, Dy, Dt)
- The Dx of a straight line is a constant (no change in slope, no max/min, no CP)