Difference Quotient (4 step method of slope)

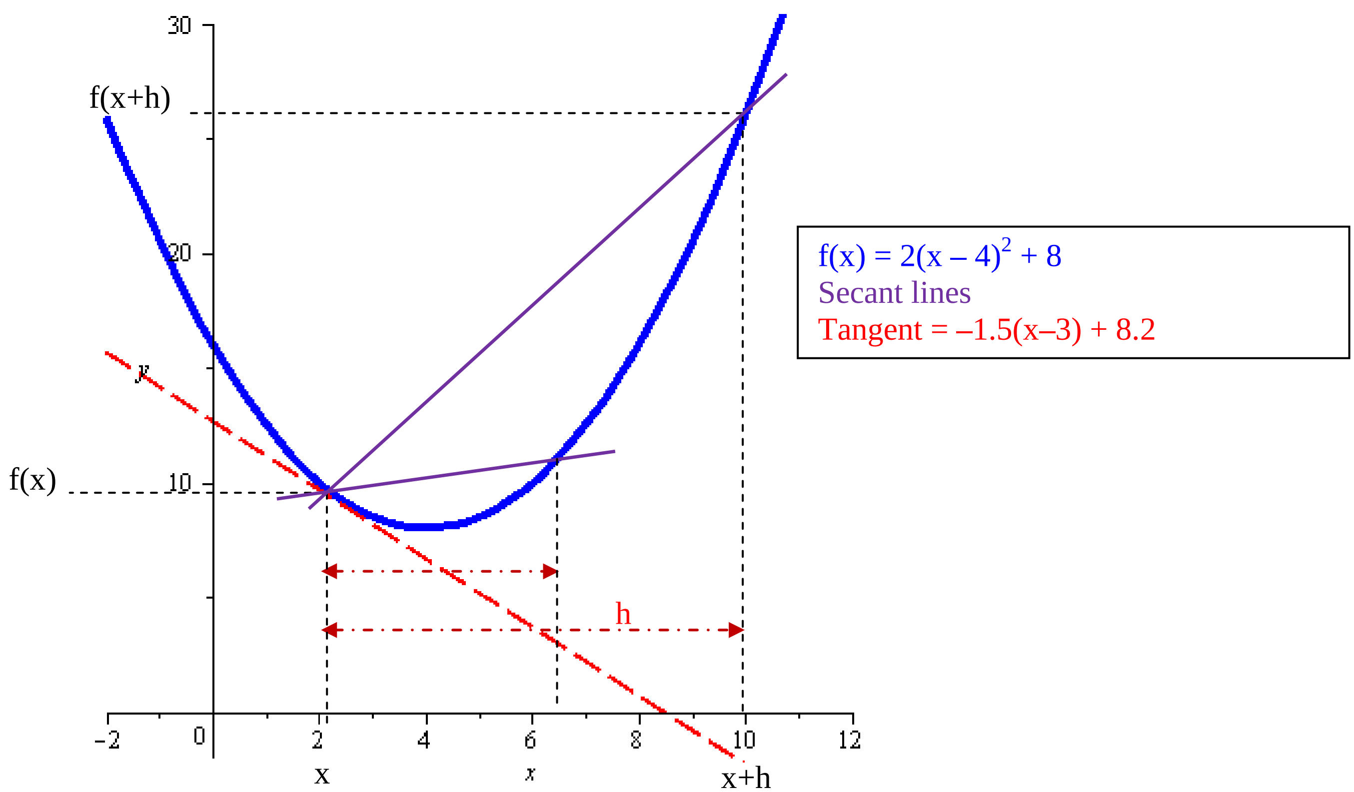
Also known as: (Definition of Limit), and (Increment definition of derivative)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

This equation is essentially the old slope equation for a line: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

***f* (x+h)**	– represents (y_2)
***f* (x)**	– represents (y_1)
x	– represents (x_1)
x + h	– represents (x_2)
h	– represents the change in x or ($x_2 - x_1$) or Δx
***f* (x+h) – *f* (x)**	– represents ($y_2 - y_1$)
Lim	– represents the slope M as h→0

$$M = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}$$



As 'h' gets smaller, the value of (x+h) gets closer to (x) and thus f(x+h) gets closer to f(x), and the slope of the secant line gets closer to the slope of the tangent line at (x). And so as h→0, we get the limit of the equation at (x)

Difference Quotient

The Difference Quotient is an algebraic approach to the *Derivative* ($\frac{dy}{dx}$) and is sometimes referred to as the "*Four Step Method.*" It is a way to find the slope of a line tangent to some function $f(x)$ at some point (x) on the function that is continuous at that (x).

The idea of a limit is to get *very close* to a given value of (x) in $f(x)$, even if $f(x)$ is not defined at (x) and so in our equation, $h \to 0$ (h approaches zero), but does not necessarily equal zero.

Process:

$f(x) = 3x^2 + 6x - 4$ ► given

Step 1: Substitute (x + h) into $f(x)$

$f(x+h) = 3(x+h)^2 + 6(x+h) - 4$ ► substitute (x+h) for every x in f(x)

$f(x+h) = 3(x^2 + 2xh + h^2) + 6(x+h) - 4$ ► expand

$f(x+h) = 3x^2 + 6xh + 3h^2 + 6x + 6h - 4$ ► remove parentheses

$f(x+h) = [3x^2 + 6x - 4] + 3h^2 + 6xh + 6h$ ► combine like terms and organize; Notice original $f(x)$ in [bracket]

Step 2: Organize terms of the Numerator (f(x+h) – f(x))

$[f(x+h)] - [f(x)] = ([3x^2 + 6x - 4] + 3h^2 + 6xh + 6h) - [3x^2 + 6x - 4]$

► assembled numerator portion

$[f(x+h)] - [f(x)] = 3h^2 + 6xh + 6h$ ► combine like terms

Step 3: Organize Difference Quotient (numerator/denominator)

$$\frac{f(x+h) - f(x)}{h} = \frac{3h^2 + 6xh + 6h}{h}$$ ► organize difference quotient

$$\frac{f(x+h) - f(x)}{h} = \frac{h(3h + 6x + 6)}{h} = \frac{3h + 6x + 6}{1}$$ ► factor out common "h"

Note: You should always be able to factor out a common 'h'

Step 4: Evaluate the Limit of the Quotient

Evaluate: $$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{3h + 6x + 6}{1}$$ ► as $h \to 0$

(3h → 0)

$$\lim \frac{f(x+h) - f(x)}{h} = \frac{6x + 6}{1} = \mathbf{6x + 6}$$ ► Lim as $h \to 0$

The slope (M) of the line tangent to $f(x) = 6x + 6$ at any given (x)

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