# Integration by Parts

Integration by Parts: $\int udv = uv - \int v\,du$

**When to use:** When you have the product of two x-terms in which one term is <u>not</u> the derivative of the other, this is the most common situation and special integrals like $\int \ln(x)\,dx$.

$$\int \underbrace{x^2}_{dv} \; \underbrace{\ln(x)}_{u}\,dx$$

> The first thing is to determine which term will be 'u', and which will be 'dv'

**Select the "u" term**: choose a term whose derivative will eventually go to zero. If neither term goes to zero, select the term that remains the same ie: $e^x$, $\ln(x)$

**Select the "dv" term**: Ideally you want the 'dv' term to be easy to integrate. Given the choice, ln(x) is <u>not</u> my first choice, because ln requires integration by parts.

$$\int x^2 \ln(x)\,dx$$

let: u = ln(x) dv = $x^2$

$du = \frac{1}{x}\,dx$ $v = \frac{x^3}{3}$

vdu

uv = $\frac{x^3}{3}\ln(x)$

$$uv - \int vdu = \frac{x^3\ln(x)}{3} - \int \frac{1}{x}\frac{x^3}{3}dx = \frac{x^3\ln(x)}{3} - \int \frac{x^2}{3}dx$$

Remembering we can pull constants outside of an integral or derivative:

$uv - \int vdu = \frac{x^3\ln(x)}{3} - \frac{1}{3}\int x^2 dx$ at this point the integral portion is a straight integral that you have already done when you integrated 'dv'

$$uv - \int vdu = \frac{x^3\ln(x)}{3} - \frac{1}{3}\frac{x^3}{3} = \frac{x^3\ln(x)}{3} - \frac{x^3}{9}$$

**Alternate approach**

Even if you do not use the below procedure, the table will help organize your terms.

The term in the "u" column will be derived each time.
The term in the "dv" column will be integrated each time.
The sign column alternates each time

Ex: $\int x^2 \sin x\,dx$

| u | dv | ± |
|---|---|---|
| $x^2$ | Sin | + |
| 2x | −cos | − |
| 2 | −sin | + |
| 0 | cos | − |

$\int v\,du$

uv

# Integration by Parts

Multiply each term along each line, including the sign:

The GREEN line gives you the "uv" term and its sign.

The RED line gives you the "$\int v\,du$" term and its sign.

$\int x^2 \ \sin x\, dx = -x^2 \cos x + \int 2x \cos x\, dx$ (note sign changes)

Since the integral portion is still an "integration by parts", you repeat the process until the integral is solved.

| u | dv | ± |
|---|---|---|
| $x^2$ | Sin | + |
| 2x | −cos | − |
| 2 | −sin | + |
| 0 | cos | − |
| 0 | −sin | + |

$\int v\,du = \int -2\sin(x)\,dx$

0

uv = 2xsin(x)

Final term = 2 cos $x$

$\int x^2 \ \sin x\, dx = -x^2 \cos x + \int 2x \cos x\, dx$

$\int x^2 \ \sin x\, dx = -x^2 \cos x \ + \ 2x \sin x - \int 2 \sin x\, dx$

$$\int x^2 \ \sin x\, dx = -x^2 \cos x + \ 2x \sin x + 2 \cos x + \ C$$

The table accounts for the changes in signs of the various terms

Special Integrals:

$\int e^u \ du = \frac{e^u}{u'}$ $\int e^{3x} = \frac{e^{3x}}{3}$

$\int \ln(x) = x\ln(x) - \ x$ (by parts)