Cost: \( C = \text{fixed cost} + \text{variable cost} \) (C= 270 + .15x) [51]

Price Demand: \( p(x) = 300 - .50x \) [51]

Revenue: \( R(x) = x[p(x)] \Rightarrow (x)(300 - .50x) = 300x - .50x^2 \) [51]

Profit: \( P = R(x) - C(x) \) [51]

**Price-Demand** (p): is usually given as some \( P(x) = \text{ax} + b \)

However, sometimes you have to create \( P(x) \) from price information.

- \( P(x) \) can be calculated using point slope equation given:
  - Price is $14 for 200 units sold. A decrease in price to $12 increases units sold to 300.
  - \( m = \frac{\Delta \text{price}}{\Delta \text{units}} = \frac{(12 - 14)}{(300 - 200)} = \frac{-2.00}{100} = -0.02 \)

  \( p(x) = m(x - x_1) + p_1 \) substitute the calculated \( m \) and one of the units \((x_1)\) and price \((p_1)\)

  \( p(x) = -0.02(x - 200) + 14 = -0.02x + 4 + 14 = -0.02x + 18 \)

**Break Even Point:** \( R(x) = C(x) \)

Where \( P(x) \) and \( R(x) \) cross. In this case there are two intersect points. Generally we are only interested in the first one where we initially break even.

**Average Cost** (\( \bar{C} \)) \( = \frac{C(x)}{x} \) is the cost per unit item [199]

**Average Price** (\( \bar{p} \)) \( = \frac{p(x)}{x} \) is the price per unit item

**Marginal (Maximum) Revenue:** \( R'(x) = \frac{d}{dx} R(x) \) solve for \( x \) at \( R'(x) = 0 \) [199]

**Marginal Cost:** \( C'(x) = \frac{d}{dx} C(x) \) solve for \( x \) at \( C'(x) = 0 \) [199]

**Marginal Profit:** \( P'(x) = \frac{d}{dx} P(x) \) solve for \( x \) at \( P'(x) = 0 \) [199]

**Marginal Average Cost:** \( \bar{C}'(x) \) [199]
Elasticity:

\[ E(p) = \frac{\left| \frac{\partial}{\partial p} p(x) \right|}{\left| \frac{\partial}{\partial p} \right|} = \frac{-p f'(p)}{f(p)} \]  

Demand as a function of price: \( x = f(p) \)

- \( E(p) = 1 \) (unit elasticity, demand change equal to price change)
- \( E(p) > 1 \) (elastic, large demand change with price)
- \( E(p) < 1 \) (inelastic, demand not sensitive to price change)

\[ x = f(p) = 10000 - 25p^2 \]

Find domain of \( p \):

\[ \text{set } f(p) \geq 0 \quad 10000 - 25p^2 \geq 0 \quad p^2 \leq 400 \quad 0 \leq p \leq 20 \]

\[ f'(p) = -50p \]

Find where \( E(p) = 1 \):

\[ E(p) = \left| \frac{-p f'(p)}{f(p)} \right| = \left| \frac{-p(-50p)}{10000 - 25p^2} \right| = \left| \frac{50p^2}{10000 - 25p^2} \right| = 1 \]

\[ 50p^2 = 10000 - 25p^2 \quad \Rightarrow \quad 75p^2 = 10000 \quad \Rightarrow \quad p^2 = 133.3 \]

\[ p = \sqrt{133.3} = 11.55 \quad \text{(remember there is no negative value for } p) \]

\[
\begin{array}{c|c|c|c}
 p & 0 & 20 \\
 E(p) & <1 & =1 & >1 \\
\end{array}
\]

Relative Rate of Change (RRC)

\[ \frac{f'(x)}{f(x)} \quad \text{(find the derivative of } f(x) \text{ and divide by } f(x)) \]

Also can be found with the \( dx \ln(f(p)) \)

Demand RRC = \[ dp \left[ \ln \left( f(p) \right) \right] = \frac{1}{x} dx \]

Price RRC = \( f(x) = 10x + 500 \)

\[ \ln f(x) = \ln [10x + 500] = \ln 10 + \ln (x + 50) \quad \text{(log expansion)} \]

\[ dx \left[ f(x) \right] = \frac{1}{x + 50} dx = \frac{1}{x + 50} \quad \text{(ln10 is a constant so } dx \ln(10) = 0) \]
Future Value of a continuous income stream:

\[ FV = e^{rT} \int_0^T f(t)e^{-rt} \, dt \]

Continuous income flow \( f(t) = 500e^{0.04t} \)
Future value: 12%
Time: 5 yrs

\[ FV = 500e^{(12)(5)} \int_0^5 e^{0.04(t)} e^{-12(t)} \, dt = 500e^{6} \left[ \frac{e^{-0.08t}}{-0.08} \right]_0^5 \]

FV = $3754

Surplus:

PS (producer’s surplus) = \( \int \bar{x} [ \bar{p} - S(x) ] \, dx \)

CS (consumer’s surplus) = \( \int_0 \bar{x} [ D(x) - \bar{p} ] \, dx \)

Equilibrium is when: PS = CS

\( \bar{x} \) is the current supply \( \bar{p} \) is the current price

The surplus is the area between the curve \( \int \bar{x} f(x) \) and the area of the box created by the equilibrium point \( \bar{x} \times f(\bar{x}) \). In Case A it is the (area of the box) – (the area under the curve); in Case B it is the (area under the curve) – (area of the box).

Gini Index: \( 2 \int_0^1 (x - f(x)) = 2 \int_0^1 x - 2 \int_0^1 f(x) \, dx \)  You can solve the integral \[416\]
of \( f(x) \) separately and then subtract it from \( 2 \int_0^1 x \) which = 1. So essentially it is \( 1 - 2 \int_0^1 f(x) \). Index is between 0 and 1.