1. **Rectangular form to polar form**

Change \( x^2 + y^2 - 2y = 0 \) to polar form

Solution:

Use: \( r^2 = x^2 + y^2 \)
and \( y = r \sin(\theta) \)

\[
x^2 + y^2 - 2y = 0 \quad \text{[Replace \( x^2 + y^2 \) with \( r^2 \)]}
\]

\[
r^2 - 2y = 0
\]

\[
r^2 - 2(\ r \sin(\theta) ) \quad \text{[replace \( y \) with \( r \sin(\theta) \)]}
\]

\[
r \ (r - 2\sin(\theta)) = 0 \quad \text{[factor out \( r \)]}
\]

We get \( r = 0 \), or \( r - 2\sin (\theta) = 0 \)

The graph of \( r = 0 \) is the pole. (It represents one point only)

The pole is included in the graph of \( r - 2\sin (\theta) = 0 \)

We can discard \( r = 0 \) and just keep

\[
r - 2\sin (\theta) = 0
\]

\[
r = 2\sin (\theta) \quad \text{[The polar form of \( x^2 + y^2 - 2y = 0 \)]}
\]

2. **Polar to Rectangular**

Change \( r = -3 \cos (\theta) \) to rectangular form

Solution:

Use: \( r^2 = x^2 + y^2 \)
and \( x = r \cos (\theta) \)

\[
r = -3 \cos (\theta) \quad \text{[Multiply by \( r \) to get \( r^2 \)]}
\]

\[
r^2 = -3r \cos (\theta) \quad \text{[Use \( r^2 = x^2 + y^2 \)]}
\]

\[
x^2 + y^2 = -3r \cos (\theta) \quad \text{[ Use \( x = r \cos (\theta) \)]}
\]
\[ x^2 + y^2 + 3x = 0 \] [Rectangular form]

\[(x^2 + 3x) + y^2 = 0 \] reorganize in \( x^2 + y^2 = r^2 \)

\[ (x^2 + 3x + \frac{9}{4}) + y^2 = 0 + \frac{9}{4} \] [complete the square]

\[ (x + \frac{3}{2})^2 + y^2 = \frac{9}{4} \] [rectangular form]

Ex: given point = 4 at 30 degree.

Convert to rectangular:

\[ y = r \sin(\theta), \quad x = r \cos(\theta) \] so \([x, y] = [4\cos(30), 4\sin(30)] = [2\sqrt{3}, 2]\]

**In general:**

Use: \( r^2 = x^2 + y^2 \)

and either \( y = r \sin(\theta) \) (when \( y \) is the term used in the original equation)

or \( x = r \cos(\theta) \) (when \( x \) is the term used in the original equation)

Sometimes it is helpful to multiply the whole equation times \( r \) as a first step, as seen above.

If you end up with \( r = \text{“some value”} \), the plot of this is just a circle with \( r \) radius.