Rules for Derivatives

Derivatives (Dx):

- In this tutorial we will use **Dx** for the derivative $(\frac{d}{dx})$. Dx indicates that we are taking the derivative with respect to x. $f'(x)$ is another symbol for representing a derivative.
- The derivative represents the slope of the function at some x, and slope represents a rate of change at that point.
- The derivative (Dx) of a constant (c) is zero.
  - Ie: y = 3 since y is the same for any x, the slope is zero (horizontal line)

Power Rule: The fundamental tool for finding the Dx of *f*(x)

- multiply the exponent times the coefficient of x and then reduce the exponent by 1

Ex: $\boldsymbol{Dx}\,[\boldsymbol{x^3}] \rightarrow f'(x) = 3x^2\,dx = 3x^2$
$\boldsymbol{Dx}\,[\boldsymbol{x^3+5}] \rightarrow f'(x) = 3x^2\,dx = 3x^2$

$$Dx\,[x^n] \xrightarrow{yields} f'(x) = nx^{n-1}$$

* [dx represents the derivative of what is inside (x), which is usually 1 for simple functions, the dx must always be considered and is always there, even if it is only 1]

Sum Rule: The Dx of a sum is equal to the sum of the Dx's

$$Dx[f(x)+\ g(x)] \Rightarrow f'(x) +\ g'(x)$$

Ex: $Dx\,[3x^2+2x+3] \rightarrow f'(3x^2) + f'(2x) +\ F'(3) = 6x+2+0$

Constant Coefficient Rule: The Dx of a variable with a constant coefficient is equal to the constant times the Dx. The constant can be initially removed from the derivation.

$$Dx[3x^2] = 3(Dx[x^2])$$

Ex: $Dx[\ln(4)\,x^2] = \ln(4)\,Dx[x^2] = \ln(4)*2x = 2\ \ln(4)\,x = \ln(4)^2 x = \ln(16)\,x$

Chain Rule: There is nothing new here other than the dx is now something other than 1. The dx represents the Dx of the inside function *g*(x). It is called a chain rule because you have to consider the dx as not being 1 and take the Dx of the inside also.

$$Dx\,[f(g(x)) \Rightarrow f'(g(x))dx = f'(g(x))g'(x)$$

Ex: Dx **(**sin**(3x))** = cos(**3x**) dx* **= 3 cos(3x)** * [dx is g'(**3x**) = 3]

* [the dx here is *g'(x)*]

Ex: Dx $[(3x^2+2)^2]$ = $2(3x^2+2)$ dx* = $2(\ 3x^2+2\ )\ (6x) = (6x^2+4)(6x) = 36x^3 + 24x$

*[dx is Dx $(3x^2+2) = 6x$] notice we used the Power Rule along with the Chain Rule

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U-sub: This is when you let some letter equal the whole inside quantity. It can be very useful in a Chain Rule situation.

Ex: Dx $[(\sin(x))^3]$ ▶ If we let $U = \sin(x)$ ⟹ then $du = \cos(x)$

Now we have: Dx $[U^3] = 3U^2du$

⟹ $3[\sin(x)]^2[\cos(x)]$ ▶ substitute back in for U and du

Product Rule: The Dx of a product is equal to the sum of the products Dx of each factor times the other factor.

$$Dx\,[f(x) \bullet g(x)] \Rightarrow [f'(x)dx \bullet g(x) + f(x) \bullet g'(x)dx]$$

Ex: $3x^2e^x = 6x(e^x) + (3x^2)e^x$

Quotient Rule: Dx (numerator) times the denominator minus Dx (denominator) times the numerator, divided by the denominator squared. This is a variation of the Product Rule.

$$Dx\left[\frac{f(x)}{g(x)}\right] \Rightarrow \frac{f'(x)g(x) - f(x)g'(x)}{[\,g(x)]^2}$$

Ex: $Dx\left[\frac{\sin(x)}{3x}\right] = \frac{\cos(x)(3x) - \sin(3x)(3)}{(3x)^2} = \frac{3x\cos(x) - 3\sin(3x)}{9x^2} = \frac{x\cos(x) - \sin(3x)}{3x^2}$

Special Rules:

- $Dx\,[\ln(x)] = \frac{1}{x\,ln(e)}dx = \frac{1}{x}$

$$Dx\,[log_b(x)_b] = \frac{1}{x ln(b)}dx$$

Ex: $Dx\,[\ln(\sin(x))] = \frac{1}{\sin(x)ln(e)}\cos(x) = \frac{\cos(x)}{\sin(x)} = \cot(x)$

Ex: $Dx\,[\log(3x^2)] = \frac{1}{(3x^2)(ln\,10)}(6x) = \frac{3}{(x)(ln\,10)}$

- $Dx\,[e^x] = e^x dx\,\ln(e)$

$$Dx\,[N^x] = N^x(\ln(N))dx$$

Ex: Dx $[3e^{4x}] = 3[(e^{4x})][\ln(e)](4)] = 12(e^{4x})$

Ex: $Dx\,[13^{x^2+5x}] = (13^{x^2+5x})[\ln(13)](2x+5)$