Logarithms: *A Logarithm is really an Exponent.* This is a fundamental idea to keep in mind when using Logs. By definition:

$$Y = \text{Log}_B(X) \quad \text{if and only if :} \quad X = B^Y$$

I use what I call the Log Loop to see this: Drawing a loop from the base (B) around through the Y to the X and read it as: B to the Y = X ($B^Y = X$).

Ex: Find X: $Log_2(32) = X$ => using the definition of a log… $2^X = 32$ therefore X = 5

A logarithm is a variation in the form of an exponential number. The two most commonly used logarithms are Base 10 and Base 'e'. Log (A) is read: Log base 10 of A. Log base 10 (Log_{10}) is referred to as the "Common" logs, whereas Log base e (Log_e) is referred to as the "Natural" logs and uses the abbreviation (Ln). Unless otherwise indicated the term Log (x) is always understood to be base 10 or $\text{Log}_{10}(x)$

Log Terminology: Base, Expand, Compress, Exponentiate, Inverse,

Log Base: There are two primary bases that are used: Base 10 (Common Log) and Base e (Natural Log). It is common practice to differentiate between them using the terms Log and Ln. The graph of a Log in any base is essentially the same; the difference being the rate of change along the curve of the graph, which means that the value obtained from the Log (A) vs Ln (A) will be different.

Expand: Expanding a Log means going from a single Log of some value to two or more Logs. This is easily understood when you look at the Multiplication Property.

$$\text{Log}(A*B) = \text{Log}(A) + \text{Log}(B)$$

You begin with a single Log of (A times B) and then expand it to the sum of two individual Logs:

$$\text{Log}(A) + \text{Log}(B)$$

We say that the original Log of (A*B) has been "expanded." The purpose of expanding, besides giving you practice in using the properties, is to allow these Logs to be further handled algebraically. As an example of this lets look at: $\text{Log}(37e^{-kt})$ By using the Multiplication and Exponent Property we can "expand" this Log to: $\text{Log}(37) + (-kt)\text{Log}(e)$

As you can see we now have a more simple algebraic statement; the exponent (-kt) has become a simple Coefficient; of course, in reality we would have used the Natural log (Ln) for this Log because of the "e" term:

$$\text{Ln}(37e^{-kt}) = \text{Ln}(37) + (-kt)\text{Ln}(e)$$

Since Ln (37) is just a number and Ln(e) = 1 ; we have: $3.6 + (-kt)$

Compress: Compressing is just going the opposite direction of Expanding; this may be as simple and taking a Log Coefficient and moving it to the exponent position.

$$3\,\text{Log}(A) = \text{Log}(A)^3$$

Exponentiate: This is also called "taking the Anti-Log" on a calculator, although on the calculator you are limited to only two bases: Base 10 and Base e. When you Exponentiate an equation you will take each term on both sides of the equation and make each of the terms an exponent of the base of the Log. Many people use the phrase "e it" because it is hard to say "exponentiate it", you can also think of this as "un-Logging it" if you want.

$\text{Log}(A) = 5$ Exponentiated → $10^{\text{Log}(A)} = 10^5$

Since Log is understood to be Base 10, unless otherwise stated, we use 10 as our exponentiation base and the Log(A) and 5 become the exponents of 10. Hence we see the why the word "exponentiate" is used.

Ex: $\text{Log}(3) + \text{Log}(5) = X$ → $10^{\text{Log}(3)} + 10^{\text{Log}(5)} = 10^X$

→ $3 + 5 = 10^X$

→ $15 = 10^X$

Note: Notice we exponentiate using whatever the base of the log is, in this case the base of the log is 10. This allows us to use the properties of logs to get a simple algebraic equation.

Properties of Logarithms:

The properties of logarithms are the key to understanding them. If you learn the properties it will make working the logs a much easier task and since there are only a few properties, it makes sense to commit them to memory.

Multiplication: $\text{Log}(A \cdot B) = \text{Log}(A) + \text{Log}(B)$

The Log of two numbers multiplied together, can be solved by taking the Log of each number and adding their Log values:

Division: $\text{Log}(A/B) = \text{Log}(A) - \text{Log}(B)$

The Log of two numbers divided together can be solved by taking the Log of each number and subtracting the Log of the denominator from the log of the numerator.

Exponent: $\text{Log}(A)^x = x\,\text{Log}(A)$ → the exp becomes a coefficient of the log

Logs are particularly useful in finding an unknown exponent in an equation like: $A = 37e^{-kt}$ By taking the Ln of both sides you get: $\text{Ln}(A) = \text{Ln}(37e^{-kt}) = \text{Ln}(37) + (-kt)\text{Ln}(e)$; now the problem is one of multiplication and division. This kind of equation occurs frequently in science and business

Fundamental Values:

$\text{Log}_B(B) = 1$ Using the Log Loop: $B^1 = B$

$\text{Ln}(e) = 1$ because $\text{Ln} = \text{Log}_e$

$\text{Log}(10) = 1$ because $\text{Log} = \text{Log}_{10}$

$\text{Log}_B(1) = 0$ Since $B^0 = 1$ > where $B \neq 0$

Ln (1) = 0
Log (1) = 0

Change of Base: The change of base formula allows you to calculate the value of any log base.

$$Log_b x = \frac{Log_a x}{Log_a b}$$

The new base is "a". If you were given a log base that was not base 10 or base e, you would not be able to find the value on your calculator. Using the base change formula you can solve for any base using a standard calculator.

$$Log_7 29 = \frac{Log_{10} 29}{Log_{10} 7} = \frac{1.4624}{.8540} = 1.7304$$