

Long Division of Polynomials

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient}$$

$$\begin{array}{r} \text{Quotient} \\ \text{Divisor} \overline{) \text{Dividend}} \end{array}$$

First, recall the terms used in division: The **dividend** is divided by the **divisor**, and the answer is the **quotient**.

$$\frac{5x^3 - x^2 + 6}{x - 4}$$

$$x - 4 \overline{) 5x^3 - x^2 + (0x) + 6}$$

1) To set up a problem involving the division of polynomials, it is best to arrange each term from highest to lowest degree and include 0 coefficients for missing degree terms. In this example, x to the third power is the highest degree and the constant 6 ($6x^0$) is the lowest. The missing term is x to the power of 1. When all the terms are arranged correctly, set up the problem with the dividend to the right of the divisor, and separate with the division bracket.

$$x - 4 \overline{) 5x^3 - x^2 + (0x) + 6}$$

$$\frac{5x^3}{x} = 5x^2$$

$$x - 4 \overline{) 5x^3 - x^2 + (0x) + 6}$$

$$5x^2(x-4) = 5x^3 - 20x^2$$

$$x - 4 \overline{) 5x^3 - x^2 + (0x) + 6} \\ \underline{-(5x^3 - 20x^2)}$$

2) To begin the division, divide the first term of the divisor into the first term of the dividend. The quotient goes above the division bracket and is aligned with the dividend term used in this step.

Next, the product of the first term of the quotient and the divisor can be taken out of the dividend. This product is notated below the original dividend and aligned with the same degree term.

This next step is important because it is where many people make their mistake: We are looking to remove what the quotient has given us so far from the dividend which means we are subtracting these terms from the original dividend. We must place a subtraction sign in front of this term with parenthesis so that all of the signs are correct.

$$x - 4 \overline{) 5x^3 - x^2 + (0x) + 6} \\ \underline{-(5x^3 - 20x^2)} \\ 19x^2 + 0x + 6$$

3) The third step is to subtract the like-polynomials. This is where coefficients are useful to hold places. The first term should be eliminated if the quotient in step 2 is correct. The strategy in subtracting out this first product is to eliminate the first term of the dividend. When the rest of the terms of the dividend are added to the difference ("brought down"), the remainder is what's left.

$$x - 4 \overline{) 5x^3 - x^2 + (0x) + 6} \\ \underline{-(5x^3 - 20x^2)} \\ 19x^2 + 0x + 6 \\ \underline{-(19x^2 - 76x)} \\ 76x + 6 \\ \underline{-(76x - 304)} \\ 310$$

Step 2 and 3 are repeated until the remainder is of a lesser degree than the divisor.

4) The **final remainder** over the divisor is added to the end of the quotient terms to give the final solution.

$$5x^2 + 19x + 76 + \frac{310}{x-4}$$