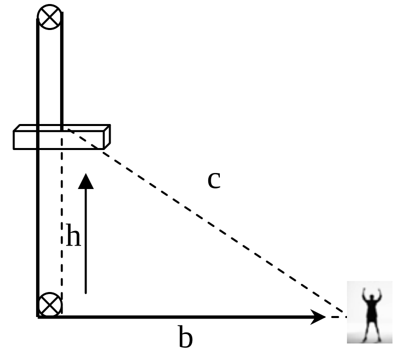
A man has a rope that goes to a pulley mounted on the floor, and up to another pulley mounted on the ceiling and down to a bale of hay. He pulls the rope by walking away from the pulley on the floor at 3 feet/second. The bale of hay is raised at a rate of 6 feet/second. He initially starts at 6 feet from the pulley with the bale of hay on the floor. Find the rate of change between the bale of hay and the man's position after 2 seconds.



Step 1: Write the given information in math notation.

t=2
dy/dt = 6 f/s
dx/dt = 3 f/s

Step 2: Write question in math notation

Rate of change of distance = derivative of distance (c)/dt = dc/dt

Step 3: Find an equation that relates the given facts to the answer

Basic formula: $c^2 = h^2 + b^2$ $\quad c = \sqrt{h^2 + b^2}$

h (height) = h(t)
b (base) = b(t)
c (distance between hay bale and man)
dc/dt = rate of change of distance between man and hay bale

Step 4: Differentiate

$$\frac{dc}{dt} = d(\sqrt{h^2 + b^2})\,dt = \frac{1}{2\sqrt{h^2 + b^2}}\left(2h\frac{dh}{dt} + 2b\frac{db}{dt}\right) \qquad \text{(implicit differentiation)}$$

Step 5: Solve for any unknowns (t=2)

$h(t) = t(\frac{dy}{dt}) = 6t$ $\qquad h(2) = 12$

$b(t) = 6 + t(\frac{dx}{dt}) = 6 + 3t$ $\qquad b(2) = 12$

$\frac{db}{dt} = \frac{dx}{dt} = 3, \quad \frac{dh}{dt} = \frac{dy}{dt} = 6$

Step 6: Substitute for solution

$$\frac{dc}{dt} = \frac{1}{2\sqrt{h^2 + b^2}}\left(2h\frac{dh}{dt} + 2b\frac{db}{dt}\right) =$$

$$\frac{1}{2\sqrt{12^2 + 12^2}}[2(12)(6) + 2(12)(3)] = \frac{216}{2\sqrt{288}} = 6.363 \text{ f/s}$$

Logic check: both the man and the bale are moving away from each other, so the rate of change will be a little more than the fastest rate (the hay bale rise rate).
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