Synthetic Division of Polynomials

Synthetic Division is considered by some to be easier and quicker. It certainly is useful for testing for roots, or evaluating an f(x) value, or just finding the quotient of polynomial division. The Divisor should be in the general form “x – r” or x = r. One advantage of synthetic division is you only use the coefficients of each term, but just as in long division you must get the terms organized by degree and substitute 0xn for any missing terms.

\[ \begin{array}{c|cccc} \text{C1} & \text{C2} & \text{C3} & \text{C4} & \ldots \\
\text{r} & \text{rC1} & \text{rCa} & \text{rCb} & \ldots \\
\text{C1} & \text{Ca} & \text{Cb} & \text{Cc} & \ldots \\
\end{array} \]

(1) The first coefficient is "brought down".
(2) The first coefficient and each subsequent sum is multiplied by r, and then carried to the next column.

The big difference in synthetic division is that you “ADD” each subsequent product and the coefficient instead of subtract, as in long division. Also the problem does not “bring down” the next term, and space-wise it is more compact.

\[ 5x^3 - 1x^2 + (0x) + 6 \div (x - 4) \]

\[ \begin{array}{c|cccc} \text{5} & -1 & 0 & 6 \\
\text{4} & 20 & 76 & 304 \\
\text{5} & 19 & 76 & 310 \\
\end{array} \]

So the answer is: \( 5x^2 + 19x + 76 + \frac{310}{x-4} \)

If the final remainder is zero, then the divisor is a root of the equation. If it is not zero, then the remainder represents f(r) with respect to the equation. This is why you can test for roots using synthetic division.

Determine if x=5 is a root of: \( x^3 - 6x^2 + x - 10 \)

\[ \begin{array}{c|cccc} \text{1} & -6 & 1 & -10 \\
\text{5} & 5 & -5 & -20 \\
\text{1} & -1 & -4 & -30 \\
\end{array} \]

No, since the last remainder is –30, not zero.

Ex: \( x^3 - 4x^2 - 11x + 30 \)

\[ \begin{array}{c|cccc} \text{1} & -4 & -11 & 30 \\
\text{5} & 5 & 5 & -30 \\
\text{1} & 1 & -6 & 0 \\
\end{array} \]

5 is a root of the equation, because the final sum is zero. The partial result at this point is: \( (x - 5)(x^2 + x - 6) \)
You can now factor further.