Synthetic Division

Synthetic division is considered a shortcut for long division of polynomials. It is used to divide polynomials of degree 2 or higher by a binomial of the form $x \quad k$. It involves the following pattern:

$$\begin{array}{c|ccc} k & a & b & c \\ & & & \\ \hline & & & \end{array}$$

Where $f(x) = ax^2 + bx + c$ and our divisor is $x \quad k$. The right side represents the polynomial and the left side represents the divisor.

Example: Consider the polynomial $f(x) = x^2 + 5x + 6$. Divide $f(x)$ by $x \quad 1$ using synthetic division.

First, set up the polynomial and divisor in the following way: $\begin{array}{c|ccc} 1 & 1 & 5 & 6 \\ & & & \\ \hline & & & \end{array}$

Next, *carry* the 1 on the polynomial side down to the bottom row: $\begin{array}{c|ccc} 1 & 1 & 5 & 6 \\ & \downarrow & & \\ \hline & \mathit{1} & & \end{array}$

Multiply the 1 on the left with the 1 in the bottom row and *carry* to the next column: $\begin{array}{c|ccc} \mathbf{1} & 1 & 5 & 6 \\ & \downarrow & \mathit{1} & \\ \hline & \mathbf{1} & & \end{array}$

Add the 5 and 1 together and write the *sum* in the bottom row: $\begin{array}{c|ccc} 1 & 1 & \mathbf{5} & 6 \\ & \downarrow & \mathbf{1} & \\ \hline & 1 & \mathit{6} & \end{array}$

Multiply the 1 on the left and the 6 in the last row and *carry* to the next column: $\begin{array}{c|ccc} \mathbf{1} & 1 & 5 & 6 \\ & \downarrow & 1 & \mathit{6} \\ \hline & 1 & \mathbf{6} & \end{array}$

Add the final column and write the *sum* in the bottom row: $\begin{array}{c|ccc} 1 & 1 & 5 & \mathbf{6} \\ & \downarrow & 1 & \mathbf{6} \\ \hline & 1 & 6 & \mathit{12} \end{array}$

Once the last column and bottom row are finished, we can interpret our answer. The number in the bottom right corner tells us the remainder. In our case, the remainder is 12. The rest of the bottom row tells us the coefficients in the quotient. In our case, the 1 and 6 tell us our quotient is $1x + 6$, or $x + 6$. Notice that the degree of the quotient is one less than the degree of $f(x)$. This gives us our final answer:

$$x + 6 + \frac{12}{x \quad 1}$$

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Theorems and Rules

Remainder Theorem: If a polynomial $f(x)$ is divided by $x \quad k$ then the remainder is $r = f(k)$.

We can check our previous example with this theorem. $f(1) = 1^2 + 5(1) + 6 = 12$, which matches the remainder we found with synthetic division.

Factor Theorem: A polynomial $f(x)$ has a factor $x \quad k$ if and only if $f(k) = 0$.

Example: Divide $x^2 \quad 5x + 6$ by $x \quad 2$

$$\begin{array}{r|rrr} 2 & 1 & -5 & 6 \\ & \downarrow & 2 & -6 \\ \hline & 1 & -3 & 0 \end{array} \qquad \frac{x^2-5x+6}{x-2} = x \quad 3$$

Let $f(x)$ be a polynomial with real coefficients and a positive leading coefficient. Suppose $x \quad k$ divides $f(x)$.
Upper Bound Rule: If $k > 0$ and each number in the bottom row is positive, then k is an upper bound for the real zeros of f.
Lower Bound Rule: If $k < 0$ and each number in the bottom row alternates between positive and negative (Note that 0 can take the place of a positive or negative), then k is a lower bound for the real zeros of f.

Example: Consider $x^3 + 3x^2 \quad 2x + 1$. Check for bounds when $x = \quad 4, \quad 2$, and 1.

Since $\quad 4 < 0$, we will check if $x = \quad 4$ is a lower bound.
Similarly, $\quad 2 < 0$, so we will check if $x = \quad 2$ is a lower bound.
Lastly, $1 > 0$, so we will check if $x = 1$ is an upper bound.

$$\begin{array}{r|rrrr} -4 & 1 & 3 & -2 & 1 \\ & \downarrow & -4 & 4 & -8 \\ \hline & 1 & -1 & 2 & -7 \end{array}$$

The bottom row follows the lower bound rule. $x = \quad 4$ is a lower bound.

$$\begin{array}{r|rrrr} -2 & 1 & 3 & -2 & 1 \\ & \downarrow & -2 & -2 & 8 \\ \hline & 1 & 1 & -4 & 9 \end{array}$$

The bottom row does not follow either rule, so it is neither.

$$\begin{array}{r|rrrr} 1 & 1 & 3 & -2 & 1 \\ & \downarrow & 1 & 4 & 2 \\ \hline & 1 & 4 & 2 & 3 \end{array}$$

The bottom row follows the upper bound rule. $x = 1$ is an upper bound.