Maximum Revenue and Profit

To find the maximum revenue and profit, we must apply the knowledge we have obtained from finding the absolute maximum and absolute minimum.

Procedure: To maximize the revenue and/or profit,

1) Determine what you are given in the word problem and derive the revenue and/or profit formula from that.
2) Take the derivative of the revenue and/or profit (in other words, the marginal revenue/profit). Set the marginal revenue/profit equal to 0. Then solve for x to find the demand.
3) Find the second derivative of the revenue and/or profit. If the second derivative is negative, then the revenue of the demand (the demand is found in step 2) should be the maximum revenue.
4) To find the maximum revenue/profit, plug the demand found in step 2 into the revenue/profit equation found in step 1.
5) (Bonus) If the question asks for the price that should be charged to get the maximum revenue/profit, plug the demand found in step 2 into the price equation that should be given.

Maximum Revenue Example: An office supply company sells x permanent markers at \$$p$ per marker. The price-demand equation for these markers is $p = 10 - 0.001x$. What price should the company charge for the markers to maximize revenue? What is the maximum revenue?

1) Determine what you have and derive the revenue formula from that.

We are given the price equation $p = 10 - 0.001x$ where x represents the demand of markers. We know from previous knowledge that the revenue $R(x)$ is equal to the price times the demand, or in other words,

$$R(x) = xp = x(10 - 0.001x) = 10x - 0.001x^2$$

2) Find the marginal revenue and set it equal to 0. Then, solve for x for the demand.

We now have the revenue, so we find the derivative of the revenue and set it equal to 0:

$$R'(x) = 10 - 0.002x$$

$$10 - 0.002x = 0$$

$$10 = 0.002x$$

$$x = 5000$$

3) To confirm that there indeed is a maximum revenue, we find the second derivative of the revenue equation:
$R''(x) = -0.002$
Since the second derivative is a negative number, there will indeed be a maximum revenue when 5000 markers are produced.
4) To find the maximum revenue/profit, plug the demand found in step 2 into the revenue/profit equation found in step 1:

$R(5000) = 10(5000) - 0.001(5000)^2 = 25000$

So, the maximum revenue will be $25000 when 5000 permanent markers are produced.

5) To find the price the company should charge to get the maximum revenue, we plug the demand into the price equation given to us:
$p = 10 - 0.001(5000) = 5$
So, to summarize our findings, the company will realize a maximum revenue of $25000 when the price of a market is $5.

Maximum Profit Example: The total annual cost of manufacturing x permanent markers for the office supply company in the previous example is $C(x) = 5000 + 2x$. What is the company's maximum profit? What should the company charge for each marker, and how many markers should be produced?

1) Determine what you are given in the word problem and derive the profit formula from that.

We are given a cost function in this problem, and we can use the revenue function that we found in the previous example. We can derive the profit function from these two functions:

$P(x) = R(x) - C(x)$

$P(x) = 10x - 0.001x^2 - (5000 + 2x) = 10x - 0.001x^2 - 5000 - 2x$

$= 8x - 0.001x^2 - 5000$

2) Find the marginal profit and set it equal to 0. Then, solve for x for the demand.

Using the profit function we just found, we find the derivative of the profit and set it equal to 0:

$P'(x) = 8 - 0.002x$

$8 - 0.002x = 0$

$8 = 0.002x$

$x = 4000$

So, the demand to get the maximum profit is 4000 markers.

3) To confirm that there indeed is a maximum profit, we find the second derivative of the profit equation:
$P''(x) = -0.002$
Since the second derivative is a negative number, there will indeed be a maximum profit when 4000 markers are produced.
4) To find the maximum profit, we plug our demand into the profit function found in step 1:
$P(4000) = 8(4000) - 0.001(4000)^2 - 5000 = 11000$
So, when 4000 markers are produced, we will have a maximum revenue of $11000.
5) To find the price that should be charged for the maximum profit, we plug the demand into the price equation from the previous example:
$p = 10 - 0.001(4000) = 6$
So, to summarize our findings, a maximum profit of $11000 is realized when 4000 markers are manufactured annually and sold for $6 each.