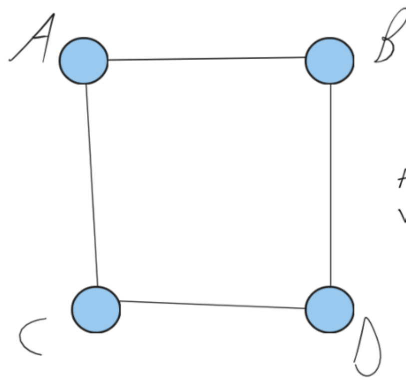
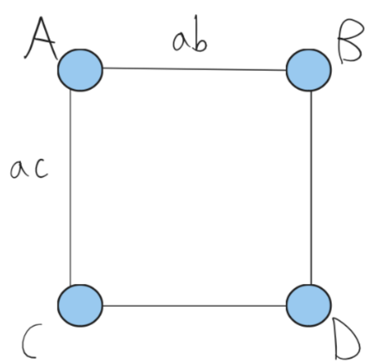
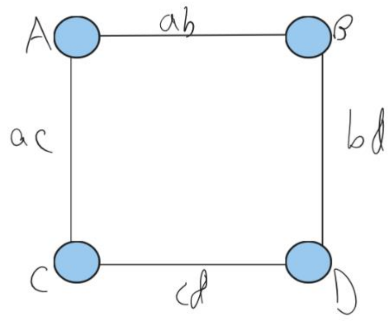
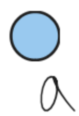
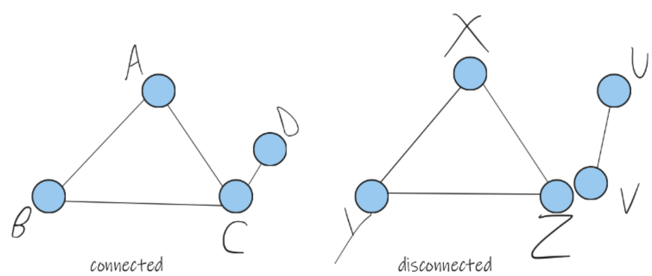
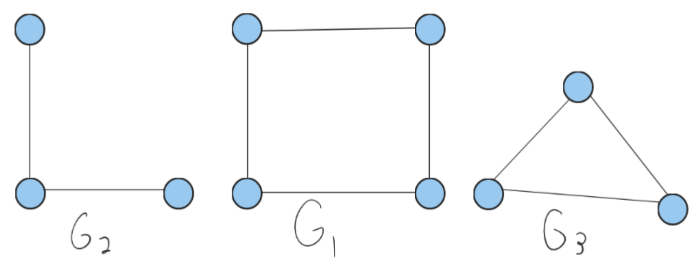
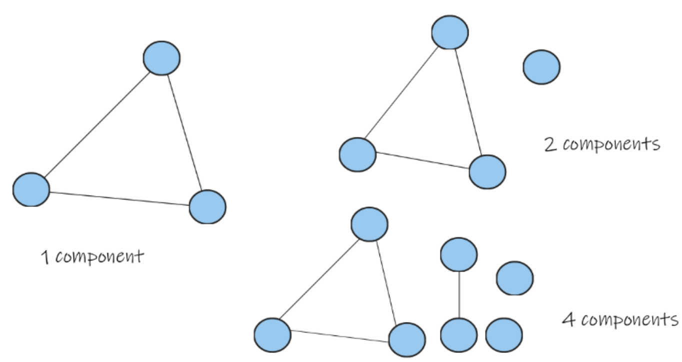
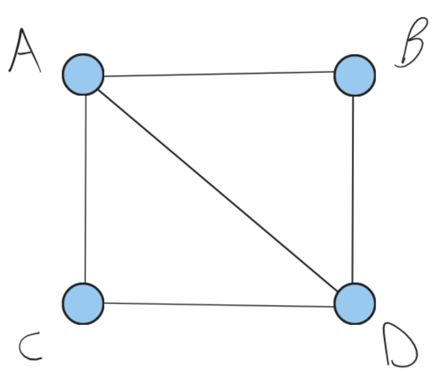
CSUSM
STEM Success Center

Graphs Handout (MATH 100 and MATH 270)

A graph G consists of a vertex set and an edge set

Definition	Example
The vertices are the dots in a graph	 A, B, C, and D are vertices
The edges are the connections between the pairs of vertices The vertices of an edge are called the endpoints	 A is incident to edges ab and ac

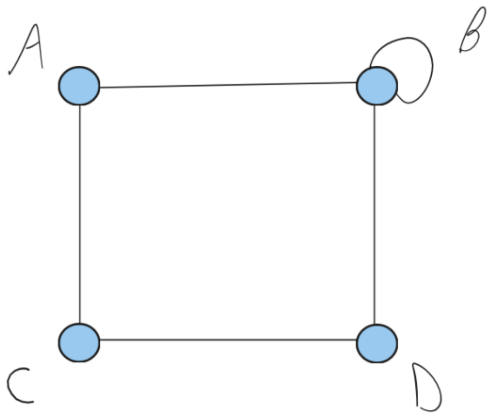
If a vertex **v** is an endpoint of an edge **e**, then **v** and **e** are incident.	 Edges: ab, ac, cd, bd (ab/ba are interchangeable, along with ac/ca, cd/dc, and bd/db)
A graph with only 1 vertex is not incident to an edge	 no incidence
A graph **G** is connected if there is a walk or path between any two vertices in **G**. A graph that is not connected is disconnected.	

A graph **H** is a subgraph of graph **G** if the vertices and edges in **H** is the same in **G**. In other words, **H** is a subgraph of **G** if there is a "copy" of **H** in **G**.	 G2 is a subgraph of G1 G3 is not a subgraph of G1
A component of a graph is a connected subgraph that is not part of any larger connected subgraph.	
The degrees of a vertex **v** is the number of edges incident to **v**.	 deg(a)=3 deg(b)=2 deg(c)=2 deg(d)=3

A loop is an edge that connects a vertex to itself. Loops contribute 2 for the degree of a vertex.

A graph has a Eulerian circuit if we can find a walk that traverses each edge exactly once and we end we started.

Euler Circuit Theorem: A connected graph has a Eulerian circuit if and only if every vertex has an even degree



deg(a)=2
deg(b)=4
deg(c)=2
deg(d)=2

All degrees are even, so the graph has a Eulerian circuit