Laws of Sine and Cosine

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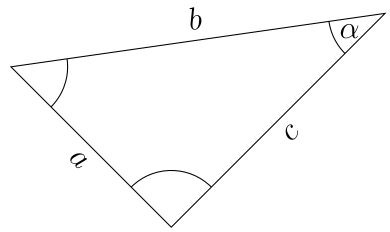
Case	Meaning	Example	Initial Law
SSA	Two sides and an angle opposite of one of them	Given a, b, and	Sines
ASA	Two angles and the side between them	Given α, , and c	Sines
AAS	Two angles and a side opposite of one of them	Given α, , and b.	Sines
SAS	Two sides and the angle between them	Given a, b, and	Cosines
SSS	Three sides	Given a, b, and c	Cosines



Law of Sines

The ratio of the sine of an angle and its opposite side is equal across all sides and sine of angles. This gives us the following:

$$\frac{\sin\alpha}{a} = \frac{\sin}{b} = \frac{\sin}{c}$$

This equation allows us to solve the cases of **SSA, ASA,** and **AAS**.

Law of Cosines

Another relationship we can use is that of cosine. We can use the following equations:

$$a^2 = b^2 + c^2 \quad 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 \quad 2ac\cos$$

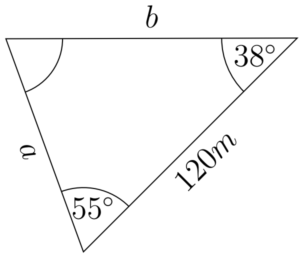
$$c^2 = a^2 + b^2 \quad 2ab\cos$$

This equation allows us to solve the cases of **SAS** and **SSS**.

NOTE: In some situations both laws may be needed.

Examples

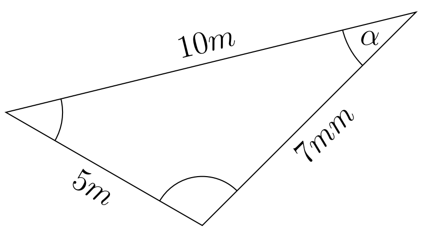
1. Solve for the missing angles and sides.



First, we can solve for   using the fact that sum of the angles in a triangle is 180°.
180   55   38 = 87. So   = 87°.
Next, we can use the law of sines to solve for the remaining sides.

$$\frac{\sin 38°}{a} = \frac{\sin 87°}{120} \implies a \approx 73.98077m$$
$$\frac{\sin 55°}{b} = \frac{\sin 87°}{120} \implies b \approx 98.43314m$$

2. Solve for the angles in the given triangle.



We can use the law of cosines to solve for the angles.

$$5^2 = 10^2 + 7^2 \quad 2(10)(7)\cos\alpha \implies \cos\alpha = \frac{25 \quad 100 \quad 49}{140}$$

$$\implies \alpha = \arccos(\frac{124}{140}) = 0.48277 \text{ radians OR } 27.7°$$

$$10^2 = 5^2 + 7^2 \quad 2(5)(7)\cos \quad \implies \cos \quad = \frac{100 \quad 25 \quad 49}{70}$$

$$\implies \quad = \arccos(\quad \frac{26}{70}) = 1.95134 \text{ radians OR } 111.8°$$

Note that the angles in a triangle add up to 180°. If we use radians, they add up to π radians. This gives us the following:

$$= \pi \quad .48277 \quad 1.95134 \approx .70748 \text{ radians OR } 40.5°$$