Inverse Trigonometric Functions

› DEFINITION 1 Inverse Sine Function

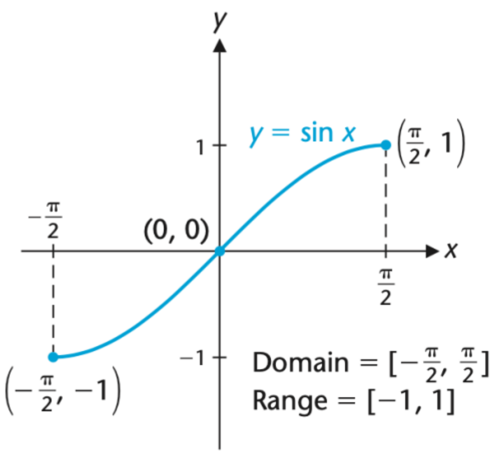
The **inverse sine function,** denoted by $\sin^{-1}$ or arcsin, is defined as the inverse of the restricted sine function $y = \sin x$, $-\pi/2 \le x \le \pi/2$. So

$$y = \sin^{-1} x \qquad \text{and} \qquad y = \arcsin x$$

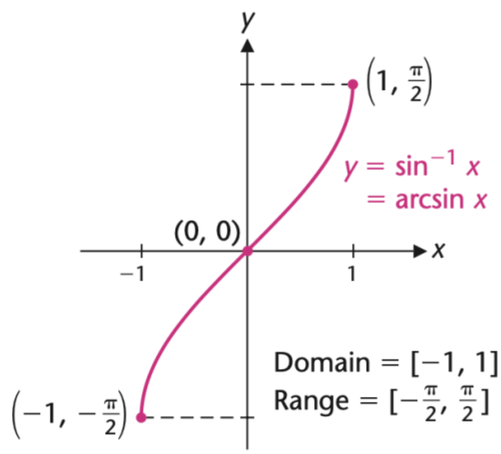
are equivalent to

$$\sin y = x \qquad \text{where} \qquad -\pi/2 \le y \le \pi/2,\ -1 \le x \le 1$$

In words, the inverse sine of x, or the arcsine of x, is the number or angle y, $-\pi/2 \le y \le \pi/2$, whose sine is x.



Restricted sine function



Inverse sine function

› SINE–INVERSE SINE IDENTITIES

$$\sin(\sin^{-1} x) = x \qquad -1 \le x \le 1 \qquad f(f^{-1}(x)) = x$$

$$\sin^{-1}(\sin x) = x \qquad -\pi/2 \le x \le \pi/2 \qquad f^{-1}(f(x)) = x$$

$\sin(\sin^{-1} 0.7) = 0.7$ $\qquad$ $\sin(\sin^{-1} 1.3) \ne 1.3$

$\sin^{-1}[\sin(-1.2)] = -1.2$ $\qquad$ $\sin^{-1}[\sin(-2)] \ne -2$

[*Note:* The number 1.3 is not in the domain of the inverse sine function, and -2 is not in the restricted domain of the sine function. Try calculating all these examples with your calculator and see what happens!]

› DEFINITION 2 Inverse Cosine Function

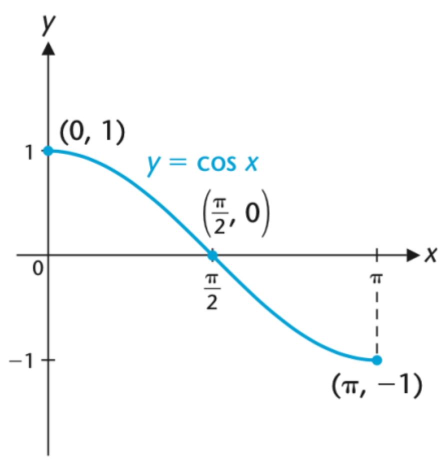
The **inverse cosine function,** denoted by $\cos^{-1}$ or arccos, is defined as the inverse of the restricted cosine function $y = \cos x$, $0 \le x \le \pi$. So

$$y = \cos^{-1} x \qquad \text{and} \qquad y = \arccos x$$

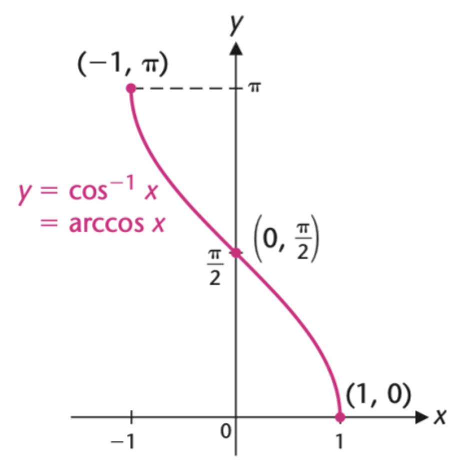
are equivalent to

$$\cos y = x \qquad \text{where} \qquad 0 \le y \le \pi,\ -1 \le x \le 1$$

In words, the inverse cosine of x, or the arccosine of x, is the number or angle y, $0 \le y \le \pi$, whose cosine is x.



Domain = [0, π]
Range = [−1, 1]
Restricted cosine function



Domain = [−1, 1]
Range = [0, π]
Inverse cosine function

› COSINE–INVERSE COSINE IDENTITIES

$\cos(\cos^{-1} x) = x$	$-1 \le x \le 1$	$f(f^{-1}(x)) = x$
$\cos^{-1}(\cos x) = x$	$0 \le x \le \pi$	$f^{-1}(f(x)) = x$

DEFINITION 3 Inverse Tangent Function

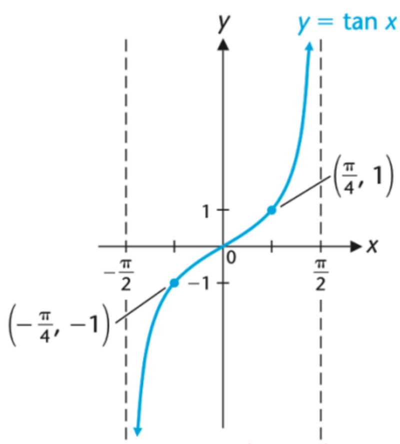
The **inverse tangent function,** denoted by $\tan^{-1}$ or arctan, is defined as the inverse of the restricted tangent function $y = \tan x$, $-\pi/2 < x < \pi/2$. So

$$y = \tan^{-1} x \qquad \text{and} \qquad y = \arctan x$$

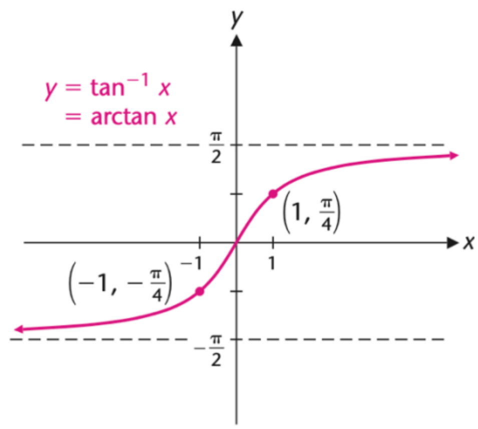
are equivalent to

$$\tan y = x \qquad \text{where} \qquad -\pi/2 < y < \pi/2 \text{ and } x \text{ is a real number}$$

In words, the inverse tangent of x, or the arctangent of x, is the number or angle y, $-\pi/2 < y < \pi/2$, whose tangent is x.



Domain = $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
Range = $(-\infty, \infty)$
Restricted tangent function



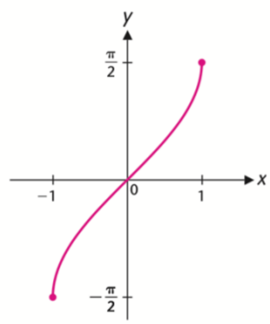
Domain = $(-\infty, \infty)$
Range = $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
Inverse tangent function

TANGENT–INVERSE TANGENT IDENTITIES

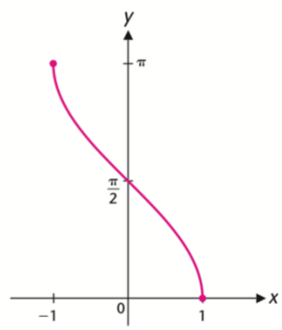
$\tan(\tan^{-1} x) = x$	$-\infty < x < \infty$	$f(f^{-1}(x)) = x$
$\tan^{-1}(\tan x) = x$	$-\pi/2 < x < \pi/2$	$f^{-1}(f(x)) = x$

SUMMARY OF $\sin^{-1}$, $\cos^{-1}$, AND $\tan^{-1}$

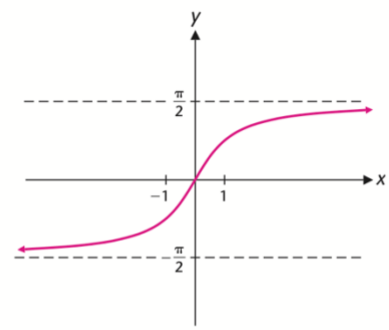
$y = \sin^{-1} x$	is equivalent to	$x = \sin y$	where $-1 \le x \le 1$, $-\pi/2 \le y \le \pi/2$
$y = \cos^{-1} x$	is equivalent to	$x = \cos y$	where $-1 \le x \le 1$, $0 \le y \le \pi$
$y = \tan^{-1} x$	is equivalent to	$x = \tan y$	where $-\infty < x < \infty$, $-\pi/2 < y < \pi/2$



$y = \sin^{-1} x$
Domain $= [-1, 1]$
Range $= [-\frac{\pi}{2}, \frac{\pi}{2}]$



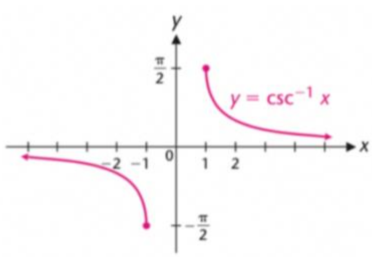
$y = \cos^{-1} x$
Domain $= [-1, 1]$
Range $= [0, \pi]$



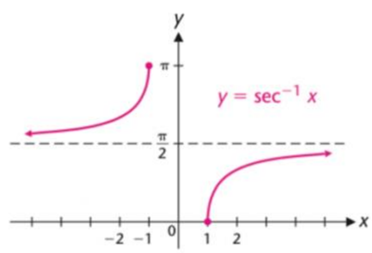
$y = \tan^{-1} x$
Domain $= (-\infty, \infty)$
Range $= \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

DEFINITION 4 Inverse Cotangent, Secant, and Cosecant Functions

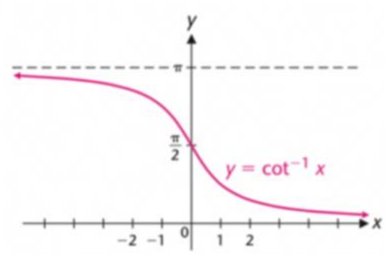
$y = \csc^{-1} x$	is equivalent to	$x = \csc y$	where $-\pi/2 \le y \le \pi/2$, $y \ne 0$, $\lvert x \rvert \ge 1$
$y = \sec^{-1} x$	is equivalent to	$x = \sec y$	where $0 \le y \le \pi$, $y \ne \pi/2$, $\lvert x \rvert \ge 1$
$y = \cot^{-1} x$	is equivalent to	$x = \cot y$	where $0 < y < \pi$, $-\infty < x < \infty$



Domain: $x \le -1$ or $x \ge 1$
Range: $-\pi/2 \le y \le \pi/2$, $y \ne 0$



Domain: $x \le -1$ or $x \ge 1$
Range: $0 \le y \le \pi$, $y \ne \pi/2$



Domain: All real numbers
Range: $0 < y < \pi$

[*Note:* The domain restrictions used in defining $\sec^{-1}$ and $\csc^{-1}$ are not universally agreed upon.]