Math 125 Chapter 10/Sections: 10-1 Topic: Systems of Linear Equations

Definition: A linear equation is often in the form of $\boldsymbol{A}x + \boldsymbol{B}y = \boldsymbol{C}z$. When there are one or more linear equations, it is known as a _____________ of linear equations

Problems I:

As the CEO of a well-known automotive manufacturer, you and your team celebrated for manufacturing a total of 7 million cars of your top two selling models. The day after, you want to know how many of each model were sold for. From your assistant, they reported that model A and model B sold for a total of $150 billion. Given that the costs of Model X and Y are $18,000 and $24,000 respectively.

Based on the description above, complete the following system of linear equations:

$$x + y = \_\_\_\_\_\_\_\_\_\_\_$$

$$\_\_\_\_\_\_\_\_ x + \_\_\_\_\_\_\_\_ y = \$150{,}000{,}000{,}000$$

From the systems of linear equations formed, find the **x** and **y** intercepts for each line:

$x + y = \_\_\_\_\_\_\_\_\_\_\_$	
x	y
0	___________
7,000,000	___________

$\_\_\_\_\_\_\_\_ x + \_\_\_\_\_\_\_\_ y = 150{,}000{,}000{,}000$	
x	y
___________	$\dfrac{150{,}000{,}000{,}000}{24{,}000}$
___________	0

Plot the above points, graph the system of linear equations, find the intersection point, check the estimate.

Problem II: From the system of linear equations formed in Problem I, use substitution to solve for **x** and **y**.

Problem III: From the system of linear equations formed in Problem I, use elimination by addition to solve for **x** and **y**.

Problem IV: Solve the following system using elimination by addition. Keep your answers in fraction form. When you are done, make sure the x, y, and z values satisfy the 3 equations.

$$12x + 2y + 3z = 97 \quad \text{E1}$$
$$-6x + 2y + 8z = 4 \quad \text{E2}$$
$$5x - 2y - 2z = 1 \quad \text{E3}$$

Math 125 Chapter 10/Sections: 10-1 Topic: Systems of Linear Equations SOLUTIONS

Definition: A linear equation is often in the form of ***A*x + *B*y = *C*z.** When there are one or more linear equations, it is known as a ____system____ of linear equations

Problems I:
As the CEO of a well-known automotive manufacturer, you and your team celebrated for manufacturing a total of 7 million cars of your top two selling models. The day after, you want to know how many of each model were sold for. From your assistant, they reported that model A and model B sold for a total of $150 billion. Given that the costs of Model X and Y are $18,000 and $24,000 respectively.

Based on the description above, complete the following system of linear equations:

$$x + y = 7{,}000{,}000$$

$$18{,}000\,x + 24{,}000\,y = 150{,}000{,}000{,}000$$

From the systems of linear equations formed, find the **x** and **y** intercepts for each line:

$x + y = 7{,}000{,}000$	
x	y
0	7,000,000
7,000,000	0

$18{,}000\,x + 24{,}000\,y = 150{,}000{,}000{,}000$	
x	y
0	$\frac{150{,}000{,}000{,}000}{24{,}000} = 6{,}250{,}000$
$\frac{150{,}000{,}000{,}000}{18{,}000} = 8{,}333{,}333$	0

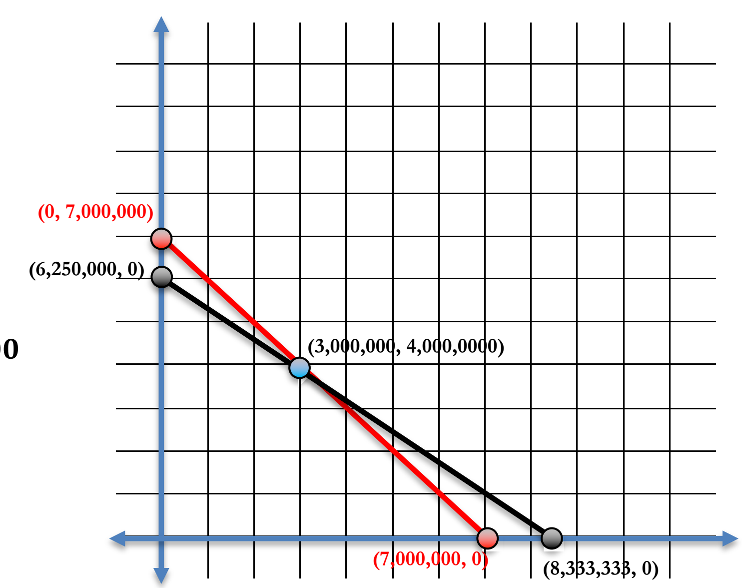
Plot the above points, graph the system of linear equations, find the intersection point, check the estimate.

x = 3,000,000
y = 4,000,000

Check:

$$\mathbf{x + y = 7{,}000{,}000}$$
$$3{,}000{,}000 + 4{,}000{,}000 = 7{,}000{,}000$$

$$\mathbf{18{,}000x + 24{,}000y = 150{,}000{,}000{,}000}$$
$$18{,}000(3{,}000{,}000) + 24{,}000(4{,}000{,}000) = 150{,}000{,}000{,}000$$



Problem II: From the system of linear equations formed in Problem I, use substitution to solve for **x** and **y**.

1. $x + y = 7{,}000{,}000$
2. $y = 7{,}000{,}000 - x$ **# Solve for y**
3. $18{,}000\,x + 24{,}000(7{,}000{,}000 - x) = 150{,}000{,}000{,}000$ **# Substitute y into the second equation**
4. $18{,}000\,x + 168{,}000{,}000{,}000 - 24{,}000\,x = 150{,}000{,}000{,}000$ **# Solve**
5. $-6{,}000\,x = 150{,}000{,}000{,}000 - 168{,}000{,}000{,}000$ **#Solve**
6. $x = -\frac{18{,}000{,}000{,}000}{-6{,}000} = 3{,}000{,}000$
7. $3{,}000{,}000 + y = 7{,}000{,}000$
8. $y = 7{,}000{,}000 - 3{,}000{,}000 = 4{,}000{,}000$
9. $x = 3{,}000{,}000$ and $y = 4{,}000{,}000$

Problem III: From the system of linear equations formed in Problem I, use elimination by addition to solve for **x** and **y**.

#Multiply by a common factor that will cancel one of the variables in the second linear equation

$$x + y = 7{,}000{,}000$$
$$(x + y = 7{,}000{,}000) * -18{,}000$$

$$(-18{,}000x - 18{,}000y = -126{,}000{,}000{,}000)$$
$$\underline{+18{,}000x + 24{,}000y = 150{,}000{,}000{,}000}$$
$$0 + 6{,}000y = 24{,}000{,}000{,}000$$
$$y = \frac{24{,}000{,}000{,}000}{6{,}000} = 4{,}000{,}000$$

$x + 4{,}000{,}000 = 7{,}000{,}000$ **# Solve for x**

$$x = 3{,}000{,}000$$
$$y = 4{,}000{,}000$$

Problem IV: Solve the following system using elimination by addition. Keep your answers in fraction form. When you are done, make sure the x, y, and z values satisfy the 3 equations.

$$12x + 2y + 3z = 97 \quad \text{E1}$$
$$-6x + 2y + 8z = 4 \quad \text{E2}$$
$$5x - 2y - 2z = 1 \quad \text{E3}$$

 # Multiply by a common factor that will cancel one of the variables in the second equation. We cancel x.

$12x + 2y + 3z = 97$ E1

$\underline{-12x + 4y + 16z = 8 \quad 2 * \text{E2}}$

$6y + 19z = 105$ E4

 # Multiply by a common factor that will cancel variable x.

$60x + 10y + 15z = 485$ $5 * \text{E1}$

$\underline{-60x + 24y + 24z = -12 \quad -12 * \text{E3}}$

$34y + 39z = 473$ E5

 # Find value of y or z. We find z.

 # Multiply by common factor

$204y + 646z = 3570$ $34 * \text{E4}$

$\underline{-204y - 234z = -2838 \quad -6 * \text{E5}}$

$412z = 732$

$$\mathbf{z = \frac{732}{412}}$$

 # Find value of y

$6y + 19\left(\frac{732}{412}\right) = 105$ E4

$y = \left(\frac{7338}{103}\right)/6$

$$\mathbf{y = \frac{\frac{7338}{103}}{6} = \frac{1223}{103}}$$

 # Find value of x

$5x - 2\left(\frac{1223}{103}\right) - 2\left(\frac{732}{412}\right) = 1$ E3

$5x = 1 + \frac{2812}{103}$

$$\mathbf{x = \frac{\frac{2915}{103}}{5} = \frac{583}{103}}$$