Math 215 Chapter 7: Trigonometric Identities and Conditional Equations/Section Topics: 1-5

Verify Identity

1. $(sinx + cosx)^2 = 1 + 2sinxcosx$

2. $\frac{cos^2x - sin^2x}{sinxcosx} = cotx - tanx$

3. $\frac{3cos^2z + 5sinz - 5}{cos^2z} = \frac{3sinz - 2}{1 + sinz}$

4. $tan(\frac{\Pi}{2} - x) = cotx$

5. $tanx - tany = \frac{sin(x-y)}{cosxcosy}$

6. $cot2x = \frac{cot^2x - 1}{2cotx}$

7. $cos^2\frac{x}{2} = \frac{1 + cosx}{2}$

8. $\frac{cosx - cosy}{sinx + siny} = -tan\frac{x-y}{2}$

Evaluate Using Appropriate Identities

9. $cos105^{\circ} - cos15^{\circ}$

10. $sin22^{\circ}cos38^{\circ} + cos22^{\circ}sin38^{\circ}$

11. $sec75^{\circ}$

Find exact solutions over the indicated intervals

12. $2cosx - 1 = 0,\ 0 \leq x \leq 2\Pi$

13. $sin^2x + 2cosx = \ -2,\ \ 0 \leq x \leq 2\Pi$

14. $2sin^2\frac{x}{2} - 3sin\frac{x}{2} + 1 = 0,\ 0 \leq x \leq 2\Pi$

15. $tanx - \sqrt{3} = 0,\ all\ real\ x$

Solutions

1. $(sinx + cosx)^2 = 1 + 2sinxcosx$ Factor left side of equation

$(sinx + cosx)(sinx + cosx) = 1 + 2sinxcosx$ Simplify

$sin^2x \ + 2sinxcosx + cos^2x = 1 + 2sinxcosx$ Simplify

$sin^2x + cos^2x + 2sinxcosx = 1 + 2sinxcosx$ Use pythagorean identity

$1 + 2sinxcosx = 1 + 2sinxcosx$ Proven equal

2. $\frac{cos^2x - sin^2x}{sinxcosx} = cotx - tanx$ Factor left side of equation

$\frac{(cosx+sinx)(cosx-sinx)}{sinxcosx} = cotx - tanx$

$\frac{cosx+sinx}{sinx} \times \frac{cosx-sinx}{cosx} = cotx - tanx$ Separated the fraction using Multiplication

$(\frac{cosx}{sinx} + \frac{sinx}{sinx}) \times (\frac{cosx}{cosx} - \frac{sinx}{cosx}) = cotx - tanx$ Separated the fractions using addition and subtraction

$(cotx + 1)(1 - tanx) = cotx - tanx$ Simplified fractions

$cotx - cotx\ tanx + 1 - tanx = cotx - tanx$ Foiled left side of equation

$cotx - 1 + 1 - tanx = cotx - tanx$ Simplified cotx tanx=1

$cotx - tanx = cotx - tanx$ Proven equal

3. $\frac{3cos^2z+5sinz-5}{cos^2z} = \frac{3sinz-2}{1+sinz}$ Use $cos^2z = 1 - sin^2z$ to substitute left side

$\frac{3(1-sin^2z)+5sinz-5}{1-sin^2z} = \frac{3sinz-2}{1+sinz}$ Simplify

$\frac{3-3sin^2z+5sinz-5}{1-sin^2z} = \frac{3sinz-2}{1+sinz}$ Simplify

$\frac{-3sin^2z+5sinz-2}{1-sin^2z} = \frac{3sinz-2}{1+sinz}$ Simplify

$\frac{-(3sinz-2)(sinz-1)}{(1+sinz)(1-sinz)} = \frac{3sinz-2}{1+sinz}$ Distribute -1 to (sinz-1)

$\frac{(3sinz-2)(-sinz+1)}{(1+sinz)(1-sinz)} = \frac{3sinz-2}{1+sinz}$ Cancel out (-sinz+1) and (1-sinz)

$\frac{3sinz-2}{(1+sinz)} = \frac{3sinz-2}{1+sinz}$ Proven equal

4. $tan(\frac{\Pi}{2} - x) = cotx$

$\frac{sin(\frac{\Pi}{2}-x)}{cos(\frac{\Pi}{2}-x)} = cotx$ Convert tan to cos and sin on left side

$\frac{sin\frac{\Pi}{2}cosx-cos\frac{\Pi}{2}sinx}{cos\frac{\Pi}{2}cosx+sin\frac{\Pi}{2}sinx} = cotx$ Used sin and cos difference identities

$\frac{cosx}{sinx} = cotx$ Simplify

$cotx = cotx$ Proven equal

5. $tanx - tany = \frac{sin(x-y)}{cosxcosy}$

$\frac{sinx}{cosx} - \frac{siny}{cosy} = \frac{sin(x-y)}{cosxcosy}$ Converted tan to sin and cos on left side

$\frac{sinxcosy}{cosxcosy} - \frac{cosxsiny}{cosxcosy} = \frac{sin(x-y)}{cosxcosy}$ Found common denominator on left side

$\frac{sinxcosy - cosxsiny}{cosxcosy} = \frac{sin(x-y)}{cosxcosy}$ Use difference identity on left side

$\frac{sin(x-y)}{cosxcosy} = \frac{sin(x-y)}{cosxcosy}$ Proven equal

6. $cot2x = \frac{cot^2x - 1}{2cotx}$

$\frac{cos2x}{sin2x} = \frac{cot^2x - 1}{2cotx}$ Converted cot to sin and cos on left side

$\frac{cos^2x - sin^2x}{2sinxcosx} = \frac{cot^2x - 1}{2cotx}$ Used double angle identities

$\frac{cos^2x - sin^2x}{2sinxcosx} = \frac{\frac{cos^2x}{sin^2x} - 1}{\frac{2cosx}{sinx}}$ Converted cot to sin and cos on right side

$\frac{cos^2x - sin^2x}{2sinxcosx} = \frac{\frac{cos^2x}{sin^2x} - \frac{sin^2x}{sin^2x}}{\frac{2cosx}{sinx}}$ Found common denominator on right side

$\frac{cos^2x - sin^2x}{2sinxcosx} = \frac{\frac{cos^2x - sin^2x}{sin^2x}}{\frac{2cosx}{sinx}}$ Simplify

$\frac{cos^2x - sin^2x}{2sinxcosx} = \frac{cos^2x - sin^2x}{sin^2x} \times \frac{sinx}{2cosx}$ Multiply by the reciprocal

$\frac{cos^2x - sin^2x}{2sinxcosx} = \frac{cos^2x - sin^2x}{2sinxcosx}$ Proven equal

7. $cos^2\frac{x}{2} = \frac{1 + cosx}{2}$

$(\pm\sqrt{\frac{1 + cosx}{2}})^2 = \frac{1 + cosx}{2}$ Use half angle identity on left side

$\frac{1 + cosx}{2} = \frac{1 + cosx}{2}$ Proven equal

8. $\frac{cosx-cosy}{sinx+siny} = -tan\frac{x-y}{2}$

$\frac{-2sin(\frac{x+y}{2})sin(\frac{x-y}{2})}{2cos(\frac{x+y}{2})sin(\frac{x-y}{2})} = -tan\frac{x-y}{2}$ Use sum to product identity on left side

$\frac{-sin(\frac{x+y}{2})}{cos(\frac{x+y}{2})} = -tan\frac{x-y}{2}$ Simplify

$-tan\frac{x-y}{2} = -tan\frac{x-y}{2}$ Proven equal

9. $cos105^\circ - cos15^\circ$

$cos(150^\circ - 45^\circ) - cos(60^\circ - 45^\circ)$

$cos150^\circ cos45^\circ + sin150^\circ sin45^\circ - (cos60^\circ cos45^\circ + sin60^\circ sin45^\circ)$

$\frac{-\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} + \frac{1}{2} \times \frac{\sqrt{2}}{2} - \frac{1}{2} \times \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}$

$-\frac{\sqrt{6}}{2}$

10. $sin22^\circ cos38^\circ + cos22^\circ sin38^\circ$ Use Sum identity

$sin(22^\circ + 38^\circ)$

$sin(60^\circ)$

$\frac{\sqrt{3}}{2}$

11. $sec75^\circ$

$\frac{1}{cos75^\circ}$

$\frac{1}{cos(45^\circ + 30^\circ)}$

$\frac{1}{cos45^\circ cos30^\circ - sin45^\circ sin30^\circ}$

$\frac{1}{\frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2}}$

$\frac{4}{\sqrt{6} - \sqrt{2}}$

12. $2cosx - 1 = 0, \ 0 \le x \le 2\Pi$ Isolate X

$2cosx = 1$

$cosx = 1/2$ Use unit circle

$x = \frac{\Pi}{3}, \frac{5\Pi}{3}$

13. $sin^2x + 2cosx = -2, \ \ 0 \le x \le 2\Pi$

$(1 - cos^2x) + 2cosx = -2$ Isolate X

$-cos^2x + 2cosx + 1 = -2$

$-cos^2x + 2cosx + 3 = 0$

$cos^2x - 2cosx - 3 = 0$

$(cosx + 1)(cosx - 3) = 0$

$cosx + 1 = 0, \ cosx - 3 = 0$

$cosx = -1, \ cosx = 3$ Use unit circle

$x = \Pi$

14. $2sin^2\frac{x}{2} - 3sin\frac{x}{2} + 1 = 0,\ 0 \le x \le 2\Pi$

$(2sin\frac{x}{2} - 1)(sin\frac{x}{2} - 1) = 0$ Factored equation

$2sin\frac{x}{2} - 1 = 0, sin\frac{x}{2} - 1 = 0$

$2sin\frac{x}{2} = 1, sin\frac{x}{2} = 1$

$sin\frac{x}{2} = \frac{1}{2},\ sin\frac{x}{2} = 1$ Use unit circle

$x = \Pi,\ x = \frac{\Pi}{3} \& \frac{5\Pi}{3}$

15. $tanx - \sqrt{3} = 0,\ all\ real\ x$

$tanx = \sqrt{3}$

$x = \frac{\Pi}{3} + \Pi n$