## Phys 201: Chapter 9 Impulse and Momentum

Before we begin the problems, it helps to list your known's and unknowns/what we are solving for. This helps with organization by identifying the what the problem is asking for. Also, red represents the solution.

Section 9.2: Solving Impulse and Momentum Problems
12) A 250 g ball collides with a wall. The figure below shows the ball's velocity and the force exerted on the ball by the wall. What is $v_{f x}$, the ball's rebound velocity?



## Known:

$m_{b}=250 \mathrm{~kg}=0.25 \mathrm{~kg}$
$v_{i x}=-\frac{10 m}{s}$
$F_{x}=500 \mathrm{~N}$ for $t=8 \mathrm{~ms}$
Find:
$v_{f x}=$ ?
Given that a force is being applied to an object during a time interval, then find the impulse $\left(\boldsymbol{J}_{\boldsymbol{x}}\right)$.

$$
J_{x}=\int_{t_{i}}^{t_{f}} \llbracket F_{x} d t=500 \int_{0 s}^{8 m s} d t=500\left[t_{f}-\triangle t_{i}\right]=500[0.008 \mathrm{~ms}-0 s]=4 \mathrm{~N} \cdot \mathrm{~s}
$$

Now, use the impulse-momentum theorem $\left(\Delta P_{x}=J_{x}\right)$ to solve for the final velocity.

$$
\begin{gathered}
\Delta P_{x}=J_{x} \\
P_{f x}-P_{i x}=J_{x} \\
m_{b} v_{f x}-m_{b} v_{i x}=J_{x}
\end{gathered}
$$

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$$
\begin{gathered}
m_{b} v_{f x}=J_{x}+m_{b} v_{i x} \\
v_{f x}=\frac{J_{x}}{m_{b}}+v_{i x} \\
v_{f x}=\frac{4 \mathrm{~N} \cdot \mathrm{~s}}{0.25 \mathrm{~kg}}+\left(-10 \frac{\mathrm{~m}}{\mathrm{~s}}\right)=16 \frac{\mathrm{~m}}{\mathrm{~s}}-10 \frac{\mathrm{~m}}{\mathrm{~s}}=6 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{gathered}
$$

## Section 9.3: Conservation of Momentum

16) A 10-m-long glider with a mass of 680 kg (including the passengers) is gliding horizontally through the air at $30 \mathrm{~m} / \mathrm{s}$ when a 60 kg skydiver drops out by releasing his grip on the glider. What is the glider's velocity just after the skydiver lets go?

## Known:

$m_{S}=60 \mathrm{~kg}$ (mass of skydiver)
$m_{G+S}=680 \mathrm{~kg}$ (mass of glider plus the mass of skydiver)
$m_{G}=m_{G+S}-m_{S}=680 \mathrm{~kg}-60 \mathrm{~kg}=620 \mathrm{~kg}$ (mass of glider)
$v_{G_{i}}=30 \frac{\mathrm{~m}}{\mathrm{~s}}$
$m_{S}=60 \mathrm{~kg}$
Note: turns out as the skydiver releases, the skydiver's final velocity will be the same as glider's initial velocity thus,
$v_{S_{f}}=v_{G_{i}}$
Find:
$v_{G_{f}}=$ ?

Using the Law of Conservation of Momentum equation ( $\boldsymbol{P}_{\boldsymbol{f}}=\boldsymbol{P}_{\boldsymbol{i}}$ ),

$$
\llbracket\left(m \rrbracket_{G}\right) v_{G_{f}}+\llbracket\left(m \rrbracket_{S}\right) v_{S_{f}} \llbracket=\left(m \rrbracket_{G+S}\right) v_{G_{i}}
$$

Substitute $v_{G_{i}}$ for $v_{S_{i}}$ given that $v_{S_{i}}=v_{G_{i}}$, then solve for the gliders final velocity

$$
\llbracket\left(m \rrbracket_{G}\right) v_{G_{f}}+\llbracket\left(m \rrbracket_{S}\right) v_{G_{i}} \llbracket=\left(m \rrbracket_{G+S}\right) v_{G_{i}}
$$

$$
\begin{gathered}
\llbracket\left(m \rrbracket_{G}\right) v_{G_{f}} \llbracket=\left(m \rrbracket_{G+S}\right) v_{G_{i}}-\llbracket\left(m \rrbracket_{s}\right) v_{G_{i}} \\
v_{G_{f}}=\frac{\llbracket\left(m \rrbracket_{G+S}\right) v_{G_{i}}-\llbracket\left(m \rrbracket_{S}\right) v_{G_{i}}}{m_{G}}=\frac{\llbracket\left(m \rrbracket_{G+S}\right)-\llbracket\left(m \rrbracket_{S}\right)}{m_{G}} \cdot\left(v_{G_{i}}\right)=\frac{680 \mathrm{~kg}-60 \mathrm{~kg}}{620 \mathrm{~kg}} \cdot\left(30 \frac{\mathrm{~m}}{\mathrm{~s}}\right) \\
=30 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{gathered}
$$

To check you answer, plug in the $v_{G_{f}}$ and all other knowns to see whether if each side are equivalent.

## Section 9.4: Inelastic Collisions

19) A 1500 kg car is rolling at $2.0 \mathrm{~m} / \mathrm{s}$. You would like to stop the car by firing a 10 kg blob of sticky clay at it. How fast should you fire the clay?

## Known:

$m_{\text {Car }}=1500 \mathrm{~kg}$
$v_{c a r}=2 \frac{\mathrm{~m}}{\mathrm{~s}}$
$m_{\text {clay }}=10 \mathrm{~kg}$

## Find:

$v_{\text {clay }}=$ ?

Note: To stop the vehicle that has momentum, the clay needs momentum of the same magnitude to make car's momentum zero.

Use the Law of Conservation of Momentum ( $\boldsymbol{P}_{\boldsymbol{f}}=\boldsymbol{P}_{\boldsymbol{i}}$ ) to solve for the clay's velocity

$$
\begin{gathered}
P_{f}=P_{i} \\
m_{\text {clay }} v_{c l a y}=m_{\text {car }} v_{c a r} \\
v_{f}=\frac{m_{\text {car }}}{m_{c l a y}}\left(v_{c a r}\right)=\frac{1500 \mathrm{~kg}}{10 \mathrm{~kg}} \cdot\left(2 \frac{\mathrm{~m}}{\mathrm{~s}}\right)=150 \mathrm{~kg} \cdot\left(2 \frac{\mathrm{~m}}{\mathrm{~s}}\right)=300 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{gathered}
$$

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To check your answer, see if the momentum of the clay is same as the momentum of the car.

## Section 9.6: Momentum in Two Dimensions

25) A 20 g ball of clay traveling east at $3.0 \mathrm{~m} / \mathrm{s}$ collides with a 30 g ball of clay traveling north at $2.0 \mathrm{~m} / \mathrm{s}$. What are the speed and the direction of the resulting 50 g ball of clay?


## Known:

$m_{1}=20 \mathrm{~g}=0.020 \mathrm{~kg}$
$\left(v_{i x}\right)_{1}=3 \frac{\mathrm{~m}}{\mathrm{~s}}$
$m_{2}=30 \mathrm{~g}=0.030 \mathrm{~kg}$
$\left(v_{i y}\right)_{2}=2 \frac{\mathrm{~m}}{\mathrm{~s}}$

## Find:

$v_{f}=$ ?
$\theta=$ ?

Use the Law of Conservation of Momentum $\left(\boldsymbol{P}_{\boldsymbol{f}}=\boldsymbol{P}_{\boldsymbol{i}}\right)$ to solve for the clay's velocity

$$
P_{f}=P_{i}
$$



X-direction: $m_{1}\left(v_{i x}\right)_{1}+m_{2}\left(v_{i x}\right)_{2}=\left(m_{1}+m_{2}\right) v_{f x}$
Rewrite $v_{f x}$ in terms of $v_{f}$ using trigonometry.

$$
\begin{gathered}
\cos (\theta)=\frac{v_{f x}}{v_{f}} \rightarrow v_{f x}=v_{f} \cos (\theta) \\
\rightarrow m_{1}\left(v_{i x}\right)_{1}+m_{2}\left(v_{i x}\right)_{2}=\left(m_{1}+m_{2}\right) v_{f} \cos (\theta) \\
\rightarrow v_{f} \cos (\theta)=\frac{m_{1}\left(v_{i x}\right)_{1}+m_{2}\left(v_{i x}\right)_{2}}{\left(m_{1}+m_{2}\right)} \\
v_{f} \cos (\theta)=1.2 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{gathered}
$$

Y-direction: $m_{1}\left(v_{i y}\right)_{1}+m_{2}\left(v_{i y}\right)_{2}=\left(m_{1}+m_{2}\right) v_{f y}$
Rewrite $v_{f y}$ in terms of $v_{f}$ using trigonometry.

$$
\begin{gathered}
\sin (\theta)=\frac{v_{f y}}{v_{f}} \rightarrow v_{f y}=v_{f} \sin (\theta) \\
\rightarrow m_{1}\left(v_{i y}\right)_{1}+m_{2}\left(v_{i y}\right)_{2}=\left(m_{1}+m_{2}\right) v_{f} \sin (\theta) \\
\rightarrow v_{f} \sin (\theta)=\frac{m_{1}\left(v_{i y}\right)_{1}+m_{2}\left(v_{i y}\right)_{2}}{\left(m_{1}+m_{2}\right)} \\
v_{f} \sin (\theta)=1.2 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{gathered}
$$

Use the Pythagorean theorem $\left(\boldsymbol{c}^{2}=\boldsymbol{a}^{2}+\boldsymbol{b}^{2}\right)$ to find $v_{f}$, thus

$$
v_{f}^{2}=v_{f x}^{2}+v_{f y}^{2}
$$

Substitute $v_{f x}$ and $v_{f y}$ (which was found above):

$$
\begin{gathered}
\rightarrow v_{f}=\sqrt{\left(v_{f} \cos (\theta)\right)^{2}+\left(v_{f} \sin (\theta)\right)^{2}} \\
v_{f}=\sqrt{\left(1.2 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}+\left(1.2 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}} \approx 1.69 \frac{\mathrm{~m}}{\mathrm{~s}} \\
\tan (\theta)=\frac{v_{f y}}{v_{f x}} \\
\rightarrow \theta=\tan ^{-1}\left(\frac{v_{f y}}{v_{f x}}\right) \\
\theta=\tan ^{-1}\left(\frac{1.2 \frac{\mathrm{~m}}{\mathrm{~s}}}{1.2 \frac{\mathrm{~m}}{\mathrm{~s}}}\right)=45^{\circ}
\end{gathered}
$$

The clay has a velocity of $1.69 \mathrm{~m} / \mathrm{s}$ heading northeast at a 45 -degree angle.

