Calculus for business 12th ed. Barnett [reference pages]

Cost: C = fixed cost + variable cost (C= 270 + .15x) [51]

Price Demand: *p(x)* = 300 – .50x [51]

Revenue: R(x) = x[*p(x)*] => (x)(300 – .50x) = 300x – .50x^2 [51]

Profit: P = Revenue (R) – Cost (C) [51]

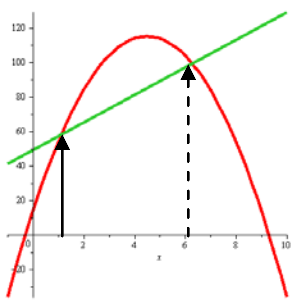
Price-Demand (p): is usually given as some P(x) = –ax + b
However, sometimes you have to create P(x) from price information.

- P(x) can be calculated using point slope equation given:
  Price is $14 for 200 units sold. A decrease in price to $12 increases units sold to 300.

$$m = \frac{\Delta\, price}{\Delta\, units} = \frac{(12-14)}{(300-200)} = \frac{-2.00}{100} = -0.02$$

p(x) = m(x – x_1) + p_1 substitute the calculated m and one of the units (**x_1**) and price (**p_1**)
p(x) = –.02(x – **200**) + **$14** = – .02x + 4 + 14 = – .02x + 18

Break Even Point: **R(x) = C(x)**



Where P(x) and R(x) cross. In this case there are two intersect points. Generally we are only interested in the first one where we initially break even.

Average Cost ($\overline{C}$) = $\frac{C(x)}{x}$ is the cost per unit item [199]

Average Price ($\overline{p}$) = $\frac{p(x)}{x}$ is the price per unit item

Marginal (Maximum) Revenue: R'(x) = $\frac{d}{dx}R(x)$ solve for x at R'(x) = 0 [199]

Marginal Cost: C'(x) = $\frac{d}{dx}C(x)$ solve for x at C'(x) = 0 [199]

Marginal Profit: P'(x) = $\frac{d}{dx}P(x)$ solve for x at P'(x) = 0 [199]

Marginal Average Cost: $\overline{C}$'(x) [199]

Elasticity: $E(p) = \left|\frac{\bar{p}}{p'}\right| = \left|\frac{p(x)}{xp'}\right| = \left|\frac{-p\,f'(p)}{f(p)}\right|$ [258]

Demand as a function of price: $x = f(p)$

$E(p) = 1$	unit elasticity (demand change equal to price change)	[259]
$E(p) > 1$	elastic (large demand change with price)	
$E(p) < 1$	inelastic (demand not sensitive to price change)	

$x = f(p) = 10000 - 25p^2$

Find domain of p: set $f(p) \geq 0$ $10000 - 25p^2 \geq 0$ $p^2 \leq 400$ $0 \leq p \leq 20$

$f'(p) = -50p$

Find where E(p) is 1:

$E(p) = \left|\frac{-pf'(p)}{f(p)}\right| = \left|\frac{-(p)(-50(p))}{10000-25(p)^2}\right| = \left|\frac{50p^2}{10000-25(p)^2}\right| = 1$

=> $50p^2 = 10000 - 25p^2$ => $75p^2 = 10000$ => $p^2 = 133.3$

$p = \sqrt{133.3} = 11.55$ (remember there is no negative value for p)

p	0			20
E(p)	<1	=1	>1	

Relative Rate of Change (RRC) [256]

$\frac{f'(x)}{f(x)}$ (find the derivative of f(x) and divide by f(x))

Also can be found with the dx(ln (f(p))

Demand RRC = dp [ln (f(p))] $dx[\ln x] = \frac{1}{x}dx$

Price RRC = $f(x) = 10x+500$

$\ln f(x) = \ln[10x+500] = \ln 10 + \ln(x+50)$ (log expansion)

$dx[f(x)] = \frac{1}{x+50}dx = \frac{1}{x+50}$ (ln10 is a constant so dx ln(10) = 0)

Future Value of a continuous income stream: **[424]**

$$FV = e^{rT} \int_0^T f(t)e^{(-rt)}\, dt$$

Continuous income flow $f(t) = 500e^{0.04t}$

Future value: 12%

Time: 5 yrs

$$FV = 500e^{(.12)(5)} \int_0^5 e^{0.04(t)}\, e^{-.12(t)}\, dt = 500e^{.6}\left[\frac{e^{-.08t}}{-.08}\right]_0^5$$

FV = $3754

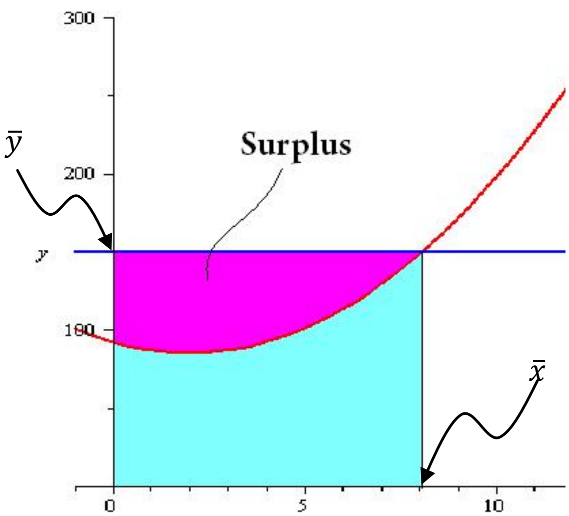
Surplus:

PS (producer's surplus) = $\int_0^{\bar{x}} [\,\bar{p} - S(x)]\, dx$ [426]

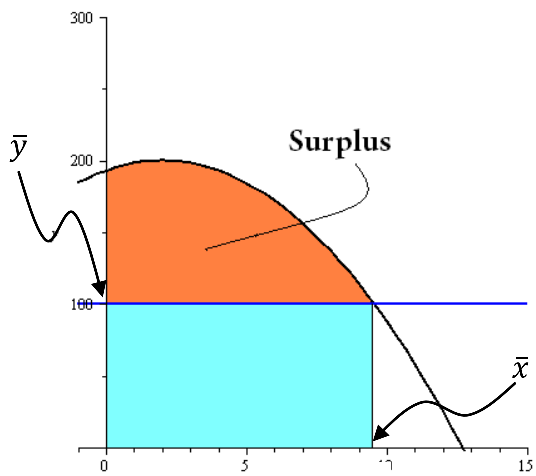
CS (consumer's surplus) = $\int_0^{\bar{x}} [\, D(x) - \bar{p}\,]\, dx$

Equilibrium is when: PS = CS

$\bar{x}$ is the current supply $\bar{p}$ is the current price



Case A



Case B

The surplus is the area between the curve [$\int_0^{\bar{x}} f(x)$] and the area of the box created by the equilibrium point (($\bar{x}\ times\ f(\bar{x})$). In Case A it is the (area of the box) – (the area under the curve); in Case B it is the (area under the curve) – (area of the box).

Gini Index: $2\int_0^1 (x - f(x)) = 2\int_0^1 x - 2\int_0^1 f(x)dx$ You can solve the integral [416] of f(x) separately and then subtract it from $2\int_0^1 x$ which = 1. So essentially it is $1 - 2\int_0^1 f(x)$. Index is between 0 and 1.