1. Rectangular form to polar form

Change $x^2 + y^2 - 2y = 0$ to polar form

Solution :

Use: $r^2 = x^2 + y^2$
and $y = r\sin(\theta)$

$x^2 + y^2 - 2y = 0$ [Replace $(x^2 + y^2)$ with r^2]

$r^2 - 2y = 0$

$r^2 - 2(r\sin(\theta))$ [replace y with $r\sin(\theta)$]

$r(r - 2\sin(\theta)) = 0$ [factor out r]

We get $r = 0$, or $r - 2\sin(\theta) = 0$

The graph of $r = 0$ is the pole. (It represents one point only)
The pole is included in the graph of $r - 2\sin(\theta) = 0$
We can discard $r = 0$ and just keep

$r - 2\sin(\theta) = 0$
$r = 2\sin(\theta)$ [The polar form of $x^2 + y^2 - 2y = 0$]

2. Polar to Rectangular

Change $r = -3\cos(\theta)$ to rectangular form

Solution:

Use: $r^2 = x^2 + y^2$
and $x = r\cos(\theta)$

$r = -3\cos(\theta)$ [Multiply by r to get r^2]

$r^2 = -3r\cos(\theta)$ [Use $r^2 = x^2 + y^2$]

$x^2 + y^2 = -3r\cos(\theta)$ [Use $x = r\cos(\theta)$]

$x^2 + y^2 + 3x = 0$	[Rectangular form]
$(x^2 + 3x) + y^2 = 0$	reorganize in $x^2 + y^2 = r^2$
$(x^2 + 3x + \frac{9}{4}) + y^2 = 0 + \frac{9}{4}$	[complete the square]
$(x + \frac{3}{2})^2 + y^2 = \frac{9}{4}$	[rectangular form]

Ex: given point = 4 at 30degree.

Convert to rectangular:

$y = r \sin(\theta)$, $x = r \cos(\theta)$ so $[x, y] = [4\cos(30), 4\sin(30)] = [2\sqrt{3}, 2]$

In general:

Use:	$r^2 = x^2 + y^2$
and either	$y = r \sin(\theta)$ (when y is the term used in the original equation)
or	$x = r \cos(\theta)$ (when x is the term used in the original equation)

Sometimes it is helpful to multiply the whole equation times r as a first step, as seen above.

If you end up with r = “some value”, the plot of this is just a circle with r radius.