The Division Algorithm: If *f(x)* and *d(x)* are polynomials such that *d(x) ≠ 0*, and the degree of *d(x)* is less than or equal to the degree of *f(x)*, then there exist unique polynomials *q(x)* and *r(x)* such that

$$f(x) = d(x)q(x) + r(x)$$

where *f(x)* is dividend, *d(x)* is divisor, *q(x)* is quotient, and *r(x)* is remainder. Also *r(x)* = 0 or the degree of *r(x)* is less than the degree of *d(x)*. If the remainder *r(x)* is zero, then *d(x)* divides evenly into *f(x)*.

Steps before applying the Division Algorithm:

1. Write the terms of the dividend and divisor in descending powers of the variable.
2. Insert placeholder with zero coefficients for missing powers of the variable.

Example 1(without remainder)**:** Divide $6x^3 - 19x^2 + 16x - 4$ by $x - 2$

$Think\ \frac{6x^3}{x} = 6x^2$

$Think\ \frac{-7x^2}{x} = -7x$

$Think\ \frac{2x}{x} = 2$

$$\begin{array}{rl} & 6x^2 - 7x + 2 \\ x - 2) & 6x^3 - 19x^2 + 16x - 4 \\ - & \underline{6x^3 - 12x^2} \\ & -7x^2 + 16x \\ - & \underline{-7x^2 + 14x} \\ & 2x - 4 \\ - & \underline{2x - 4} \\ & 0 \end{array}$$

Multiply: *6x²(x-2)*

Subtract and bring down *+16x*

Multiply: *-7x(x-2)*

Subtract and bring down *-4*

Multiply: *2(x-2)*

Subtract

From the division, you have shown that $6x^3 - 19x^2 + 16x - 4 = (x - 2)(6x^2 - 7x + 2)$ and by factoring the quadratic $6x^2 - 7x + 2$, you have

$6x^3 - 19x^2 + 16x - 4 = (x - 2)(2x - 1)(3x - 2)$

Example 2 (with remainder)**:** Divide $x^2 + 3x + 5$ by $x + 2$

$$
\begin{array}{rll}
 & x + 2 & \leftarrow \text{Quotient} \\
x + 1 \,\big) & \overline{x^2 + 3x + 5} & \leftarrow \text{Dividend} \\
- & \underline{x^2 + x} & \\
 & 2x + 5 & \\
- & \underline{2x + 2} & \\
 & 3 & \leftarrow \text{Remainder}
\end{array}
$$

So, the result is

$$\frac{x^3 + 3x + 5}{x + 1} = x + 2 + \frac{3}{x + 1}$$

If we multiply both sides by *x + 1,* then we get the answer which illustrates the Division Algorithm Theorem above:

$$x^2 + 3x + 5 = (x + 1)(x + 2) + 3$$

Example 3 (without remainder)**:** Divide $x^3 - 1$ by $x - 1$

Since there is no ***x²-term*** or ***x-term*** in the dividend $x^3 - 1$, you need to rewrite the dividend as $x^3 + 0x^2 + 0x - 1$

$$
\begin{array}{rll}
 & x^2 + x + 1 & \\
x - 1 \,\big) & \overline{x^3 + 0x^2 + 0x - 1} & \\
- & \underline{x^3 - x^2} & \text{Multiply: } x^2(x - 1) \\
 & x^2 + 0x & \text{Subtract and bring down } 0x \\
- & \underline{x^2 - x} & \text{Multiply: } x(x - 1) \\
 & x - 1 & \text{Subtract and bring down } -1 \\
- & \underline{x - 1} & \text{Multiply: } 1(x - 1) \\
 & 0 & \text{Subtract}
\end{array}
$$

So, the result is

$$\frac{x^3 - 1}{x - 1} = x^2 + x + 1, \;\; x \neq 1$$

Example 4 (with remainder in linear form)**:** Divide $-5x^2 - 2 + 3x + 2x^4 + 4x^3$ by $2x - 3 + x^2$

Write the terms of the dividend and divisor in descending powers of *x*.

$$\begin{array}{r l}
 & 2x^2 \qquad\quad + \;\; 1 \\
x^2 + 2x - 3 & \overline{)\,2x^4 + 4x^3 - 5x^2 + 3x - 2} \\
- & \underline{\;\;2x^4 + 4x^3 - 6x^2} \\
 & \qquad\qquad\quad x^2 + 3x - 2 \\
 & \qquad\quad - \;\; \underline{x^2 + 2x - 3} \\
 & \qquad\qquad\qquad\quad x + 1
\end{array}$$

Multiply: $2x^2(x^2 + 2x - 3)$

Subtract and bring down $3x - 2$

Multiply: $1(x^2 + 2x - 3)$

Subtract

Notice that after each subtraction we must have three terms. Also, observe that the first subtraction eliminated two terms from the dividend. When this happens, the quotient skips a term. So, the result is

$$\frac{2x^4 + 4x^3 - 5x^2 + 3x - 2}{x^2 + 2x - 3} = 2x^2 + 1 + \frac{x + 1}{x^2 + 2x - 3}$$