California State University SAN MARCOS

ALGEBRA

Radical Rules

For all of the following n is an integer and **n ≥ 2**.

Definitions	**Examples**
$b = \sqrt[n]{a}$ if both $b \geq 0$ and $b^n = a$	$\sqrt[3]{8} = 2$ because $2^3 = 8$
If n is odd then $\sqrt[n]{a^n} = a$	$\sqrt[7]{(-5)^7} = -5$ $\sqrt[5]{x^5} = x$
If n is even then $\sqrt[n]{a^n} = \|a\|$	$\sqrt{(-5)^2} = \|-5\| = 5$ $because \sqrt{(-5)^2} = \sqrt{25} = 5$ $\sqrt[4]{x^4} = \|x\|$
If $a \geq 0$ then $\sqrt[n]{a^n} = a$	$\sqrt[5]{\pi^5} = \pi$ $\quad$ $\sqrt[4]{\pi^4} = \pi$

Distributing ($a \geq 0$ and $b \geq 0$)	**Examples**
$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$	$\sqrt[4]{48} = \sqrt[4]{16} \cdot \sqrt[4]{3} = 2\sqrt[4]{3}$
$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ $(b \neq 0)$	$\sqrt[3]{\frac{8}{125}} = \frac{\sqrt[3]{8}}{\sqrt[3]{125}} = \frac{2}{5}$
$\sqrt[n]{a} \cdot \sqrt[n]{a} \cdot \sqrt[n]{a} \cdot \ldots \cdot \sqrt[n]{a} = a$ (when $\sqrt[n]{a}$ multiplied by itself n times)	$\sqrt[3]{4} \cdot \sqrt[3]{4} \cdot \sqrt[3]{4} = 4$ because $\sqrt[3]{4} \cdot \sqrt[3]{4} \cdot \sqrt[3]{4} = \sqrt[3]{4^3} = 4$ following the definition $\sqrt[n]{a^n} = a$
$\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}}$ $\quad$ $(m \geq 0)$	$\sqrt[6]{2^3} = 2^{\frac{3}{6}} = 2^{\frac{1}{2}} = \sqrt{2}$

Rationalizing the Denominator *(a > 0, b > 0, c > 0)*	**Examples**
$\frac{a}{\sqrt[n]{b}} = \frac{a}{\sqrt[n]{b}} \cdot \frac{\sqrt[n]{b}}{\sqrt[n]{b}} \cdot \ldots = \frac{a}{\sqrt[n]{b}} \cdot \frac{\sqrt[n]{b^{n-1}}}{\sqrt[n]{b^{n-1}}} =$ $= \frac{a\sqrt[n]{b^{n-1}}}{\sqrt[n]{b^n}} = \frac{a\sqrt[n]{b^{n-1}}}{b}$	$\frac{16}{\sqrt[4]{2}} = \frac{16}{\sqrt[4]{2}} \cdot \frac{\sqrt[4]{2}}{\sqrt[4]{2}} \cdot \frac{\sqrt[4]{2}}{\sqrt[4]{2}} \cdot \frac{\sqrt[4]{2}}{\sqrt[4]{2}} = \frac{16}{\sqrt[4]{2}} \cdot \frac{\sqrt[4]{2^3}}{\sqrt[4]{2^3}} =$ $= \frac{16\sqrt[4]{2^3}}{\sqrt[4]{2^4}} = \frac{16\sqrt[4]{8}}{2} = 8\sqrt[4]{8}$
$\frac{a}{\sqrt[n]{b^m}} = \frac{a}{\sqrt[n]{b^m}} \cdot \frac{\sqrt[n]{b^{n-m}}}{\sqrt[n]{b^{n-m}}}$	$\frac{2}{\sqrt[5]{9}} = \frac{2}{\sqrt[5]{3^2}} \cdot \frac{\sqrt[5]{3^3}}{\sqrt[5]{3^3}} = \frac{2\sqrt[5]{27}}{\sqrt[5]{3^5}} = \frac{2\sqrt[5]{27}}{3}$
$\frac{a}{b-\sqrt{c}} = \frac{a}{b-\sqrt{c}} \cdot \frac{b+\sqrt{c}}{b+\sqrt{c}} = \frac{a(b+\sqrt{c})}{b^2-(\sqrt{c})^2} =$ $= \frac{a(b+\sqrt{c})}{b^2-c}$	$\frac{5}{3-\sqrt{2}} = \frac{5}{3-\sqrt{2}} \cdot \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{5(3+\sqrt{2})}{3^2-2} =$ $= \frac{5(3+\sqrt{2})}{7}$

Remember!!! *(common errors)*	**Examples**
$\sqrt[n]{a+b} \neq \sqrt[n]{a} + \sqrt[n]{b}$	$\sqrt[3]{2+6} \neq \sqrt[3]{2} + \sqrt[3]{6}$ but $\sqrt[3]{2+6} = \sqrt[3]{8} = 2$
$\sqrt[n]{a-b} \neq \sqrt[n]{a} - \sqrt[n]{b}$	$\sqrt[3]{7-5} \neq \sqrt[3]{7} - \sqrt[3]{5}$ but $\sqrt[3]{7-5} = \sqrt[3]{2}$
$\sqrt[n]{a^n \pm b^n} \neq a \pm b$	$\sqrt[3]{2^3+3^2} \neq 2+3 \neq 5$ $\sqrt[3]{2^3+3^2} = \sqrt[3]{8+9} = \sqrt[3]{17}$