



Equations: ① $f(x) = \frac{4x^2 + 3x + 2}{4x - 1}$

② $f(x) = \frac{2x}{x - 2}$

③ $f(x) = \frac{2x}{\sqrt{x - 5}}$

Note: The above numbered equations will be referenced below using the same notation e.g. ①

Domain depends on numerator and denominator

We begin with the real numbers interval $(-\infty, \infty)$ then look for numbers or intervals to exclude:

- 1) Numbers that cause only the denominator to be zero
- 2) Numbers that cause an even root to be negative (whether its on the denominator or numerator)
- 3) Numbers that cause a root of the denominator to be zero

Example: ③ Look at the denominator: $\sqrt{x - 5} = 0$ when $x = 5$. Observe that $x < 5$ causes $(x - 5) < 0$. Values under the even root must be positive therefore domain = $(5, \infty)$

Vertical, Horizontal, and Oblique Asymptotes

Vertical Asymptote: ① Set the denominator equal to 0. You get the following:

$$f(x) = 4x - 1 = 0 \Leftrightarrow 4x = 1 \Leftrightarrow x = \frac{1}{4}$$

By plugging in $x = \frac{1}{4}$ for the numerator, you see that the numerator does not equal zero.

The above step is needed to ensure that $x = \frac{1}{4}$ is not a removable discontinuity.

Therefore, $x = \frac{1}{4}$ is a vertical asymptote.

Horizontal Asymptote: The limit method may be used to find all horizontal asymptotes. ① The degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote.

② Taking the limit as the x of numerator and denominator head to positive and negative infinity, we get that $y = 2$.

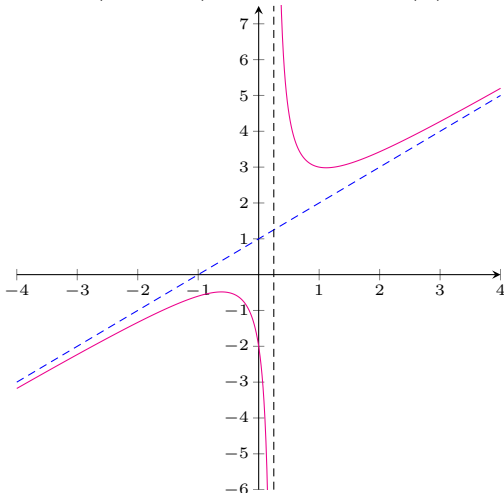
Oblique Asymptote: ① Divide the numerator by the denominator if the highest degree of the numerator is greater than the highest degree of the denominator.

$$f(x) = \frac{8x^2 + 6x + 4}{8x - 2} = \frac{2(4x^2 + 3x + 2)}{2(4x - 1)}$$

Long division of the above equation is shown below. Division stops when the remainder's highest degree is less than the divisor. In this case, $8x - 2$ is the divisor. The oblique asymptote is $y = x + 1$.

$$\begin{array}{r} x + 1 \\ 8x - 2 \overline{) 8x^2 + 6x + 4} \\ \underline{- 8x^2 + 2x} \\ 8x + 4 \\ \underline{- 8x + 2} \\ 6 \end{array}$$

The following function has a vertical and oblique (slanted) asymptote: $f(x) = \frac{8x^2 + 6x + 4}{8x - 2}$



The following function has a vertical and horizontal asymptote: $f(x) = \frac{2x}{x - 2}$

