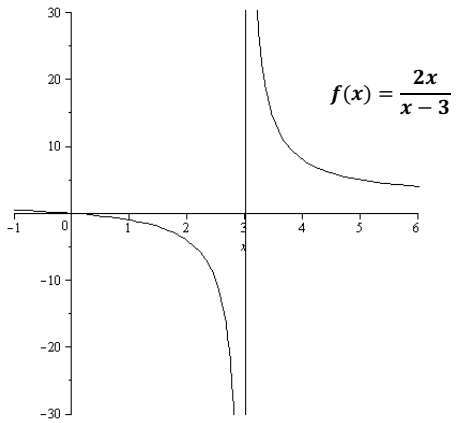
LIMITS CALCULUS

DEFINITION: We say that the limit of $f(x)$ equals L as x approaches a, written as:

$$\lim_{x\to a} f(x) = L$$

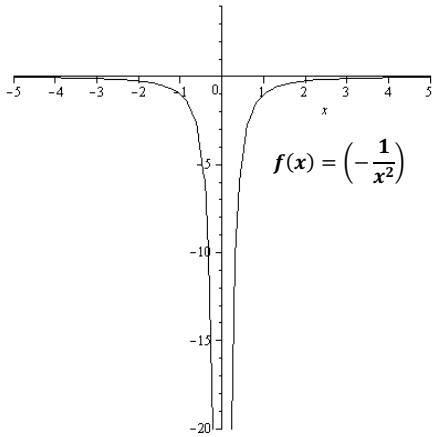
If we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a (on either side of a) but not equal to a.

THEOREM: $\lim_{x\to a} f(x) = L$ **if and only if** $\lim_{x\to a+} f(x) = L$ **and** $\lim_{x\to a-} f(x)$



$\lim_{x\to 3-} f(x) = -\infty \neq \lim_{x\to 3+} f(x) = +\infty$

The Limit Does Not Exist



$\lim_{x\to 0-} f(x) = -\infty = \lim_{x\to 0+} f(x)$

Thus, $\lim_{x\to 0} f(x) = -\infty$

Limit Laws: Suppose c is a constant and that $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist

$\lim_{x\to a} [f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$

$\lim_{x\to a} [f(x)]^n = [\lim_{x\to a} f(x)]^n$

$\lim_{x\to a} cf(x) = c \lim_{x\to a} f(x)$

$\lim_{x\to a} x^n = a^n$

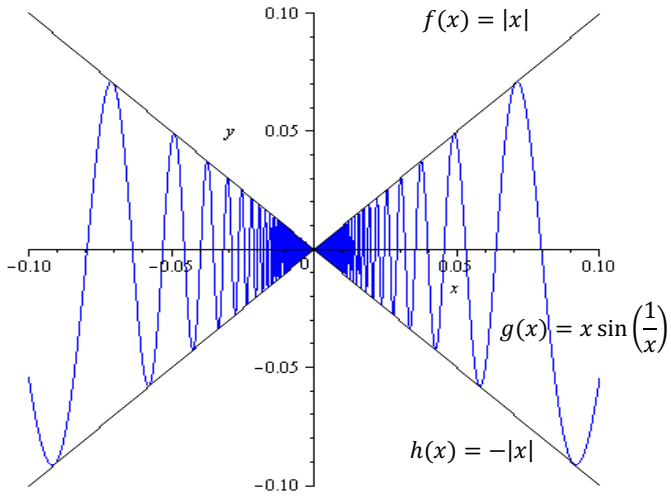
$\lim_{x\to a} f(x)g(x) = \lim_{x\to a} f(x) \lim_{x\to a} g(x)$

$\lim_{x\to a} c = c$

$\lim_{x\to a} \left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ $for\ g(x) \neq 0$

SQUEEZE THEOREM: $If\ f(x) \leq g(x) \leq h(x), when\ x\ is\ near\ a, and \lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L,\ then \lim_{x\to a} g(x) = L$

Example 1(Using Squeeze Theorem):



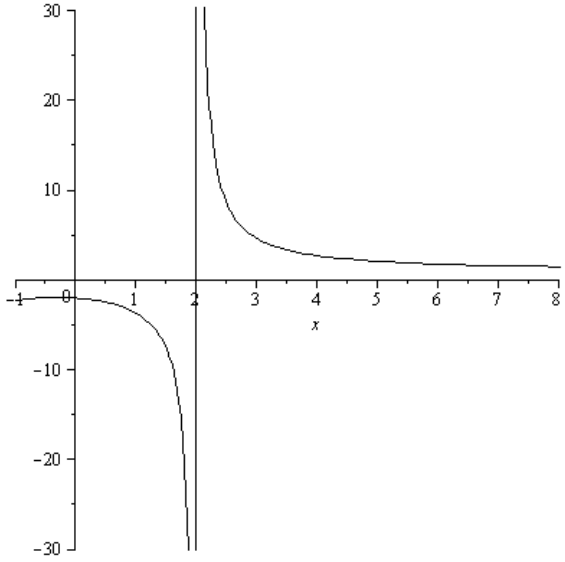
Observation for example 2:

Find $\lim_{x\to a+} f(x)$ *and* $\lim_{x\to a-} f(x)$

By observing the behavior of x as it approaches a from both the left and right hand sides, we can calculate the limit.

$\lim_{x\to 2+} \frac{x^2+2x+8}{x^2-4} = \infty$ $\lim_{x\to 2-} \frac{x^2+2x+8}{x^2-4} = -\infty$

Example 2:



x	$f(x)$	x	$f(x)$
1	-3.66667	3	4.6
1.8	-17.4211	2.5	10.7778
1.9	-38.9673	2.1	40.5122
1.99	-399.499	2.01	400.501
1.9999	40008.5	2.0001	39991.5

$\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$	$\lim_{x\to a} x = a$
$\lim_{x\to a} f(x)g(x) = \lim_{x\to a} f(x) \lim_{x\to a} g(x)$	$\lim_{x\to\infty}\left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ *for* $g(x) \neq 0$
$\lim_{x\to a}[f(x)]^n = [\lim_{x\to a} f(x)]^n$	$\lim_{x\to a} x^n = a^n$
$\lim_{x\to a} cf(x) = c \lim_{x\to a} f(x)$	$\lim_{x\to a} c = c$