



# Calculus II

## Series - Things to Consider

**Important:** This cheat sheet is not intended to be a list of guaranteed rules to follow. This introduces some hints and some ideas you may consider when choosing tests for convergence or divergence when evaluating a given series.

It is usually a good idea to try using the **Test for Divergence** as a first step when evaluating a series for convergence or divergence. If we can show that:

$$\lim_{n \rightarrow \infty} a_n \neq 0$$

Then we can say that the series diverges without having to do any extra work.

Below are some general cases in which each test may help:

### P-Series Test:

- The series be written in the form:  $\sum \frac{1}{n^p}$

### Geometric Series Test:

- When the series can be written in the form:  $\sum a_n r^{n-1}$  or  $\sum a_n r^n$

### Direct Comparison Test:

- When the given series,  $a_n$  looks like a known, or more simple, series,  $b_n$

### Limit Comparison Test:

- When you can see that the series looks like another convergent or divergent series,  $b_n$
- But it is hard to say whether  $b_n > a_n$  or  $b_n < a_n$

### Root Test:

- When the series can be written in the form:  $\sum (a_n)^n$

### Alternating Series Test:

- When the series can be written in the form:  $\sum (-1)^{n+1} a_n$  or  $\sum (-1)^n a_n$

### Ratio Test:

- Whenever we are given something involving a factorial, e.g.  $n!$
- Whenever we are given something involving a constant raised to the  $n^{\text{th}}$  power, e.g.  $\sum \frac{n+5}{5^n}$

### Integral Test:

- If the sequence is:
  - continuous
  - positive
  - decreasing (we can use the First Derivative Test here)





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**Remember:** These are just suggestions. There are other tests which may get us to the same answer.

