Probability of an Event Occurring

$$Probablity = \frac{\#\ of\ Favorable\ Outcomes}{Total\ \#\ of\ Outcomes}$$

Probability of an Event NOT Occurring

$$Probablity\ of\ Event\ NOT\ Occurring = 1 - Probability\ of\ Event\ Occurring$$

Disjunction

$$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$

Theoretical vs. Empirical Probability

Theoretical Probability: Theoretical probability is given by the number of ways a specific event can occur divided by the total number of outcomes. In general, it is used to predict what should happen.

Empirical Probability: Empirical Probability is given by the number of times a particular event occurs, divided by the number of observed incidents. In general, it is used when looking at what actually happened.

Sensitivity vs. Specificity

Sensitivity: Sensitivity is the probability of a true positive.

$$Sensitivity = \frac{True\ Positives}{All\ Who\ Have\ the\ Disease}$$

Specificity: Specificity is the probability of a true negative.

$$Specificity = \frac{True\ Negatives}{All\ Who\ do\ NOT\ Have\ the\ Disease}$$

Predictive Value

Positive Predictive Value (PPV): Probability a person who tests positive actually has the disease.

$$PPV = \frac{True\ Positives}{All\ Positives}$$

Negative Predictive Value (NPV): Probability a person who tests negative really does not have the disease.

$$NPV = \frac{True\ Negatives}{All\ Negatives}$$

Tel:
STEM SC (N): (760) 750-4101
STEM SC (S): (760) 750-7324

Conditional Probability

Conditional Probability is the probability that one event occurs given that another has occurred. Sensitivity, specificity, and predictive value are all conditional probabilities.

$$P(B \text{ given } A) = \frac{P(A \text{ and } B)}{P(A)}$$

Example Suppose you draw a card from a deck of cards. What is the probability that the card you draw is a heart given that it is a red card?

$$P(Heart \text{ given } Red) = \frac{P(Red \text{ and } Heart)}{P(Red)} = \frac{\frac{13}{52}}{\frac{26}{52}} = \frac{13}{26} = \frac{1}{2}$$

The Counting Principle

The Counting Principle demonstrates how to calculate the results of multiple experiments performed in succession. Suppose there are N outcomes for the first experiment and M outcomes for the second. If there are M outcomes of the second experiment for each outcome in the first experiment, we have the following formula (which can be extended for more experiments if needed):

$$N \times M$$

Example Suppose someone asked you for a 3-digit passcode of only numbers. There are 10 numbers to choose from $(0 - 9)$ for each digit.

$$10 \times 10 \times 10 = 10^3 = 1000$$

So there are 1000 different possible passcodes.

Note: The counting principle can be used with the probability formula.

Example What is the probability that the 3-digit passcode begins with a 1?
By the above formula we have $1 \times 10 \times 10 = 100$, i.e. 100 passcodes start with a 1. We have 1000 total outcomes and 100 favorable outcomes.

$$P(Passcode \text{ } Starts \text{ } with \text{ } 1) = \frac{100}{1000} = \frac{1}{10} = .1 = 10\%$$

Independent Events

Two events are considered independent if knowing that one event occurs has no effect on the probability that the other occurs. One can use the following equation to check for independence:

$$P(B \text{ given } A) = P(B)$$

If the equation holds true then A and B are independent events, otherwise they are dependent.
Example (recall our conditional probability example)

$$P(Heart \text{ given } Red) = \frac{1}{2} \neq \frac{1}{4} = P(Heart) \quad \textbf{Dependent}$$

Product Formula for Independent Events

Knowing that two events are independent allows us to use the product formula for independent events.

$$P(A \text{ and } B) = P(A) \times P(B)$$

Permutations

A permutation of items is an arrangement of items in a certain **order**. Order matters for permutations. We use the following to count the number of ways to select and order k items from a list of n items:

$$_nP_k \quad or \quad P(n,k) = \frac{n!}{(n-k)!}$$

Example How many ways can a president, vice president, and treasurer be chosen from a group of 10 people?

We have 10 total people (our $\boldsymbol{n}$) and 3 different positions (our $\boldsymbol{k}$).

$$P(10,3) = \frac{10!}{(10-3)!} = \frac{10!}{7!} = 10 \times 9 \times 8 = 720$$

Combinations

A combination of items is a selection of items from a group. Order does **NOT** matter with combinations. We use the following to count the number of ways to select k items from a list of n items:

$$_nC_k \quad or \quad \binom{n}{k} \quad or \quad C(n,k) = \frac{n!}{k!(n-k)!}$$

Example Suppose 8 people enter a raffle where there are 3 identical prizes. How many different combinations of winners are possible?

We proceed similarly to our previous example.

$$C(8,3) = \frac{8!}{3!(8-3)!} = \frac{8!}{3!5!} = \frac{8 \times 7 \times 6 \times 5!}{3!5!} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$$

Using Permutations or Combinations for Probability

We can extend our use of permutations and combinations to probability.

Example Suppose I want to select a 4-player team at random from a group of 10 people, where there are 6 men and 4 women. What is the probability the 4-player team is all male?

$$\frac{\#\ of\ All\ Male\ Team}{Total\ \#\ of\ Combinations} = \frac{C(6,4)}{C(10,4)} = \frac{\left(\frac{6!}{4!(6-4)!}\right)}{\left(\frac{10!}{4!(10-4)!}\right)} = \frac{15}{210} = \frac{1}{14}$$

Expected Value

Expected Value is the return you can expect for some type of action performed. It is usually used to measure the amount expected to win or lose per play of a game, in the long run. The following formula is used to calculate the expected value:

$$Expected\ Value = P(A) \times (Profit/Loss\ from\ A) + P(B) \times (Profit/Loss\ from\ B)$$

Note: The formula can be extended if there are more than 2 outcomes

Example Suppose you play a game where you pick a marble from a bag. The bag contains 1 blue marble, 2 red marbles, and 1 green marble. You pay \$1 to play. If you pick blue you win \$2 back, if you pick red you win nothing back, and if you pick green you win \$1 back. What is the game's expected value?

First we calculate the probability of each event:

$P(Blue) = \frac{1}{4}$, $P(Red) = \frac{2}{4} = \frac{1}{2}$, and $P(Green) = \frac{1}{4}$

Next we use the Expected Value Formula. Note that because we pay \$1 initially, we have to take that into account for our profit. For example, winning \$2 after paying \$1 to play results in a profit of \$1.

$$Expected\ Value = P(Blue) \times (\$1) + P(Red) \times (-\$1) + P(Green) \times (\$0)$$
$$Expected\ Value = \frac{1}{4} \times (\$1) + \frac{1}{2} \times (-\$1) + \frac{1}{4} \times (\$0)$$
$$Expected\ Value = \$0.25 + (-\$0.50) + (\$0) = -\$0.25$$

The game's expected value is -\$0.25. In the long run, you can expect to lose about 25 cents per play.

Law of Large Numbers

According to the law of large of numbers, you will almost certainly win/lose approximately what the theoretical expected value of the game is in the long run.

Gambler's Fallacy

The gambler's fallacy is the belief that a series of losses in the past will be compensated by wins in the future. A series of losses does not guarantee wins will follow.

Definition of Fair

A game is considered fair if the expected value is 0.