California State University SAN MARCOS

TRIGONOMETRY LAWS AND IDENTITIES

QUOTIENT IDENTITIES

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$
$$\cot(x) = \frac{\cos(x)}{\sin(x)}$$

RECIPROCAL IDENTITIES

$$\csc(x) = \frac{1}{\sin(x)} \qquad \sin(x) = \frac{1}{\csc(x)}$$
$$\sec(x) = \frac{1}{\cos(x)} \qquad \cos(x) = \frac{1}{\sec(x)}$$
$$\cot(x) = \frac{1}{\tan(x)} \qquad \tan(x) = \frac{1}{\cot(x)}$$

EVEN/ODD IDENTITIES

$$\sin(-x) = -\sin(x) \qquad \csc(-x) = -\csc(x)$$
$$\cos(-x) = \cos(x) \qquad \sec(-x) = \sec(x)$$
$$\tan(-x) = -\tan(x) \qquad \cot(-x) = -\cot(x)$$

PYTHAGOREAN IDENTITIES

$$\cos^2(x) + \sin^2(x) = 1$$
$$\tan^2(x) + 1 = \sec^2(x)$$
$$\cot^2(x) + 1 = \csc^2(x)$$

SUM IDENTITIES

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$
$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$
$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$$

DIFFERENCE IDENTITIES

$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$
$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$
$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$$

LAW OF SINES

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

DOUBLE-ANGLE IDENTITIES

$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos(2x) = \cos^2(x) - \sin^2(x)$$
$$= 2\cos^2(x) - 1$$
$$= 1 - 2\sin^2(x)$$
$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$

HALF-ANGLE IDENTITIES

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos(x)}{2}}$$
$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos(x)}{2}}$$
$$\tan\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos(x)}{1+\cos(x)}}$$

PRODUCT TO SUM IDENTITIES

$$\sin(x)\sin(y) = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$$
$$\cos(x)\cos(y) = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$$
$$\sin(x)\cos(y) = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$$
$$\cos(x)\sin(y) = \frac{1}{2}[\sin(x+y) - \sin(x-y)]$$

SUM TO PRODUCT IDENTITIES

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\sin(x) - \sin(y) = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$
$$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

LAW OF COSINES

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$
$$b^2 = a^2 + c^2 - 2ac\cos(B)$$
$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

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DEFINITIONS

$$\sin(x) = \frac{Opposite}{Hypotenuse} \qquad \csc(x) = \frac{Hypotenuse}{Opposite}$$

$$\cos(x) = \frac{Adjacent}{Hypotenuse} \qquad \sec(x) = \frac{Hypotenuse}{Adjacent}$$

$$\tan(x) = \frac{Opposite}{Adjacent} \qquad \cot(x) = \frac{Adjacent}{Opposite}$$