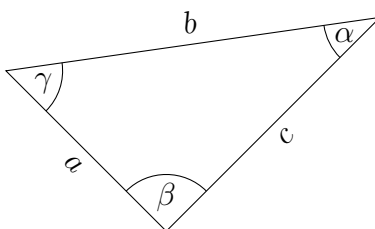




Case	Meaning	Example	Initial Law
SSA	Two sides and an angle opposite of one of them	Given $a$ , $b$ , and $\beta$	Sines
ASA	Two angles and the side between them	Given $\alpha$ , $\beta$ , and $c$	Sines
AAS	Two angles and a side opposite of one of them	Given $\alpha$ , $\beta$ , and $b$ .	Sines
SAS	Two sides and the angle between them	Given $a$ , $b$ , and $\gamma$	Cosines
SSS	Three sides	Given $a$ , $b$ , and $c$	Cosines



## Law of Sines

The ratio of the sine of an angle and its opposite side is equal across all sides and sine of angles. This gives us the following:

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

This equation allows us to solve the cases of **SSA**, **ASA**, and **AAS**.

## Law of Cosines

Another relationship we can use is that of cosine. We can use the following equations:

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

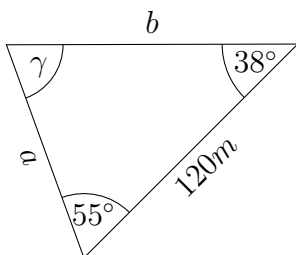
This equation allows us to solve the cases of **SAS** and **SSS**.

**NOTE: In some situations both laws may be needed.**



## Examples

1. Solve for the missing angles and sides.



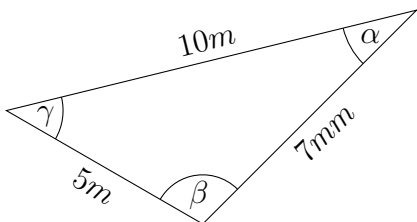
First, we can solve for  $\gamma$  using the fact that sum of the angles in a triangle is  $180^\circ$ .  
 $180 - 55 - 38 = 87$ . So  $\gamma = 87^\circ$ .

Next, we can use the law of sines to solve for the remaining sides.

$$\frac{\sin 38^\circ}{a} = \frac{\sin 87^\circ}{120} \implies a \approx 73.98077m$$

$$\frac{\sin 55^\circ}{b} = \frac{\sin 87^\circ}{120} \implies b \approx 98.43314m$$

2. Solve for the angles in the given triangle.



We can use the law of cosines to solve for the angles.

$$5^2 = 10^2 + 7^2 - 2(10)(7) \cos \alpha \implies \cos \alpha = \frac{25 - 100 - 49}{-140}$$

$$\implies \alpha = \arccos\left(\frac{-124}{-140}\right) = 0.48277 \text{ radians OR } 27.7^\circ$$

$$10^2 = 5^2 + 7^2 - 2(5)(7) \cos \beta \implies \cos \beta = \frac{100 - 25 - 49}{-70}$$

$$\implies \beta = \arccos\left(-\frac{26}{70}\right) = 1.95134 \text{ radians OR } 111.8^\circ$$

Note that the angles in a triangle add up to  $180^\circ$ . If we use radians, they add up to  $\pi$  radians. This gives us the following:

$$\gamma = \pi - .48277 - 1.95134 \approx .70748 \text{ radians OR } 40.5^\circ$$