

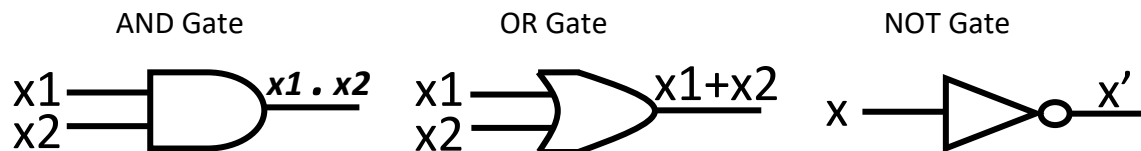
Introduction to Digital Circuits 2

Truth Table:

- The operations of a logic circuit can be defined by what is called a truth table.
- A truth table lists all the possible combinations of the input variables and shows the relationship between the input variables and the resulting output.
- They grow exponentially in size with the number of variables. A truth table with three input variables has eight rows, 2^3 since there are eight possible valuations of these variables. For four-input variables the truth table has 16 rows, 2^4 , and so on.

x_1	x_2	$x_1 * x_2$	$x_1 + x_2$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

Types of Logic Gates:



Boolean Algebra:

- To simplify a function and design a less costly circuit and more efficient circuit we use Boolean Algebra.

If assuming Boolean algebra only takes one of two values, 0 or 1, then the following is true:

1a) $0 * 0 = 0$	1b) $1 + 1 = 1$	
2a) $1 * 1 = 1$	2b) $0 + 0 = 0$	
3a) $0 * 1 = 1 * 0 = 0$	3b) $1 + 0 = 0 + 1 = 1$	

$$4a) \quad \text{If } x = 0, \text{ then } x = 1$$

$$4b) \quad \text{If } x = 1, \text{ then } x = 0$$

If assuming Boolean algebra takes one or more variables, then the following terms are true:

$$1a) \quad x + 0 = x$$

$$1b) \quad x \cdot 1 = x$$

$$2a) \quad x + x' = 1$$

$$2b) \quad x \cdot x' = 0$$

$$3a) \quad x + x = x$$

$$3b) \quad x \cdot x = x$$

$$4a) \quad x + 1 = 1$$

$$4b) \quad x \cdot 0 = 0$$

$$5a) \quad (x')' = x$$

$$\text{Commutative: } a) \quad x + y = y + x$$

$$b) \quad xy = yx$$

$$\text{Associative: } a) \quad x + (y + z) = (x + y) + z$$

$$b) \quad x(yz) = (xy)z$$

$$\text{Distributive: } a) \quad x(y + z) = xy + xz$$

$$b) \quad x + yz = (x + y) \cdot (x + z)$$

$$\text{DeMorgan: } a) \quad (x + y)' = x' \cdot y'$$

$$b) \quad (xy)' = x' + y'$$

$$\text{Absorption: } a) \quad x + xy = x$$

$$b) \quad x(x + y) = x$$

DeMorgan's Law:

- The **dual** of an expression is obtained by replacing all addition operators with multiplication operators, and vice versa, and by replacing all 0s with 1s, and vice versa. (DeMorgan law)
- Example:
Find the complement of the functions $F1 = x'yz' + x'y'z$ and $F2 = x(y'z' + yz)$ by applying DeMorgan's theorem as many times as necessarily

$$\begin{aligned} F1' &= (x'yz' + x'y'z)' \\ &= (x'yz')'(x'y'z)' \\ &= (x + y' + z)(x + y + z') \end{aligned}$$

$$\begin{aligned} F2' &= [x(y'z' + yz)]' \\ &= x' + (y'z' + yz)' \\ &= x' + (y'z')' \cdot (yz)' \\ &= x' + (y + z)(y' + z') \end{aligned}$$

Procedures to represent a function in Sum of minterms and product of maxterms:

- To find the sum of product of a given function from truth table (**SoP**):
 1. Make the truth table for the function
 2. Look at those rows that function is 1
 3. Write down the corresponding product terms and sum them together to find sum of minterms
- To find the product of sums (**Pos**):
 1. From the truth table for the function, f
 2. Find the SoP of the complement of the function, f' (use the terms whose functional values are 0)
 3. find out $(f)'$ which will result in f but with product of maxterms

Example of SoP and Pos:

Given the function $f = AB + A'C$, find its Representation in sum of minterm and product of maxterm.

1. Make the truth table for function by putting the value of function to 1 for those terms that $AB=1$ or $A'C = 1$
2. Finding the **sum of minterms** form of function:

$$f = A'B'C + A'BC + ABC' + ABC$$
3. Find the complement of function by summing the minterms that are 0 in the function.

$$f' = A'B'C' + A'BC' + AB'C' + AB'C$$
4. Complement f' one more time and the result would be f in terms of **Product of maxterm**

$$(f')' = f = (A+B+C)(A+B'+C)(A'+B+C)(A'+B+C)$$

ABC	AB	A'C	f
000	0	0	0
001	0	1	1
010	0	0	0
011	0	1	1
100	0	0	0
101	0	0	0
110	0	0	1
111	1	0	1

