**Inverse Trigonometric Functions**

### › DEFINITION 1 Inverse Sine Function

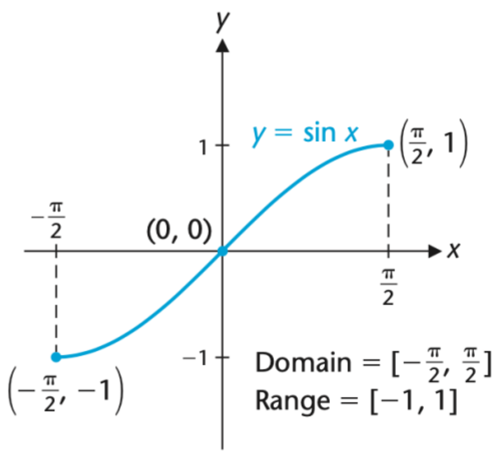
The **inverse sine function,** denoted by $\sin^{-1}$ or arcsin, is defined as the inverse of the restricted sine function $y = \sin x$, $-\pi/2 \le x \le \pi/2$. So

$$y = \sin^{-1} x \qquad \text{and} \qquad y = \arcsin x$$

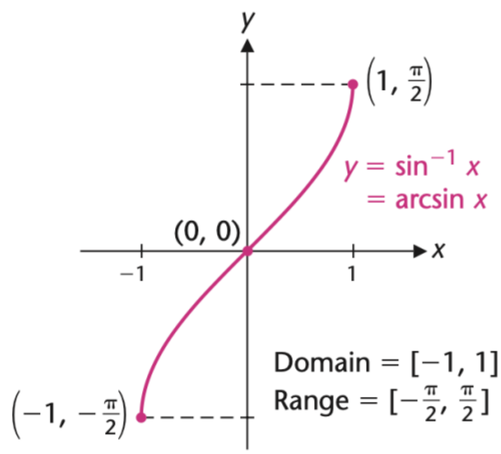
are equivalent to

$$\sin y = x \qquad \text{where} \qquad -\pi/2 \le y \le \pi/2,\ -1 \le x \le 1$$

In words, the inverse sine of $x$, or the arcsine of $x$, is the number or angle $y$, $-\pi/2 \le y \le \pi/2$, whose sine is $x$.




Restricted sine function




Inverse sine function

### › SINE–INVERSE SINE IDENTITIES

$\sin(\sin^{-1} x) = x \qquad -1 \le x \le 1 \qquad f(f^{-1}(x)) = x$

$\sin^{-1}(\sin x) = x \qquad -\pi/2 \le x \le \pi/2 \qquad f^{-1}(f(x)) = x$

$\sin(\sin^{-1} 0.7) = 0.7 \qquad \sin(\sin^{-1} 1.3) \ne 1.3$

$\sin^{-1}[\sin(-1.2)] = -1.2 \qquad \sin^{-1}[\sin(-2)] \ne -2$

[*Note:* The number 1.3 is not in the domain of the inverse sine function, and $-2$ is not in the restricted domain of the sine function. Try calculating all these examples with your calculator and see what happens!]

### DEFINITION 2 Inverse Cosine Function

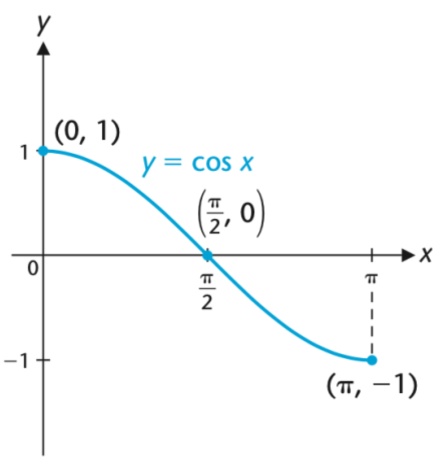
The **inverse cosine function,** denoted by $\cos^{-1}$ or arccos, is defined as the inverse of the restricted cosine function $y = \cos x$, $0 \le x \le \pi$. So

$$y = \cos^{-1} x \qquad \text{and} \qquad y = \arccos x$$

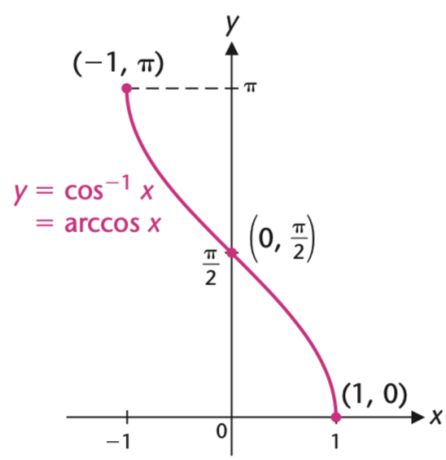
are equivalent to

$$\cos y = x \qquad \text{where} \qquad 0 \le y \le \pi, \ -1 \le x \le 1$$

In words, the inverse cosine of $x$, or the arccosine of $x$, is the number or angle $y$, $0 \le y \le \pi$, whose cosine is $x$.




Domain = $[0, \pi]$
Range = $[-1, 1]$
Restricted cosine function




Domain = $[-1, 1]$
Range = $[0, \pi]$
Inverse cosine function

### COSINE–INVERSE COSINE IDENTITIES

$$\cos(\cos^{-1} x) = x \qquad -1 \le x \le 1 \qquad f(f^{-1}(x)) = x$$

$$\cos^{-1}(\cos x) = x \qquad 0 \le x \le \pi \qquad f^{-1}(f(x)) = x$$

### DEFINITION 3 Inverse Tangent Function

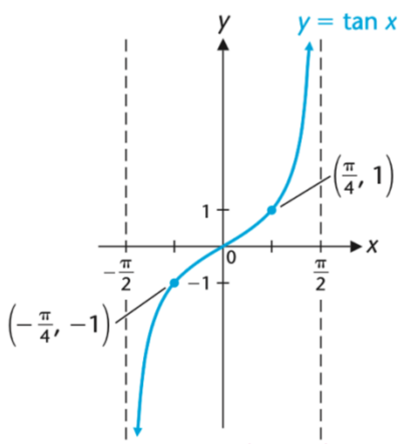
The **inverse tangent function,** denoted by $\tan^{-1}$ or arctan, is defined as the inverse of the restricted tangent function $y = \tan x$, $-\pi/2 < x < \pi/2$. So

$$y = \tan^{-1} x \qquad \text{and} \qquad y = \arctan x$$

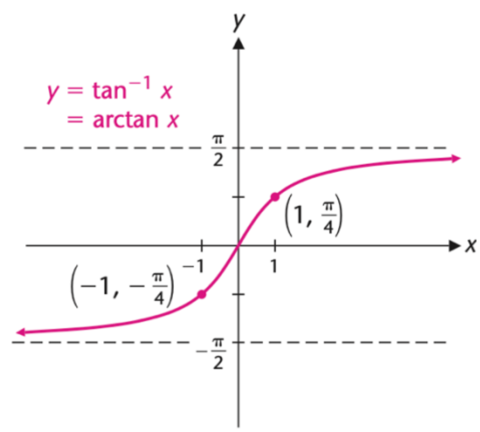
are equivalent to

$$\tan y = x \qquad \text{where} \qquad -\pi/2 < y < \pi/2 \text{ and } x \text{ is a real number}$$

In words, the inverse tangent of $x$, or the arctangent of $x$, is the number or angle $y$, $-\pi/2 < y < \pi/2$, whose tangent is $x$.




Domain $= \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
Range $= (-\infty, \infty)$
Restricted tangent function



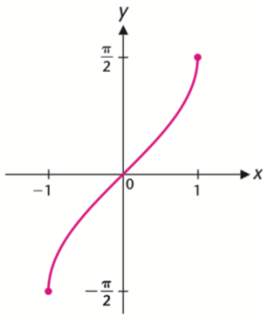

Domain $= (-\infty, \infty)$
Range $= \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
Inverse tangent function

### TANGENT–INVERSE TANGENT IDENTITIES

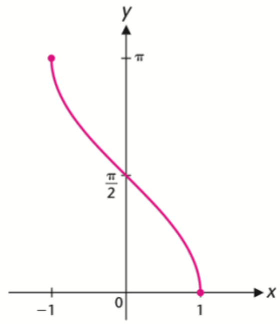
| | | |
|---|---|---|
| $\tan(\tan^{-1} x) = x$ | $-\infty < x < \infty$ | $f(f^{-1}(x)) = x$ |
| $\tan^{-1}(\tan x) = x$ | $-\pi/2 < x < \pi/2$ | $f^{-1}(f(x)) = x$ |

## SUMMARY OF $\sin^{-1}$, $\cos^{-1}$, AND $\tan^{-1}$

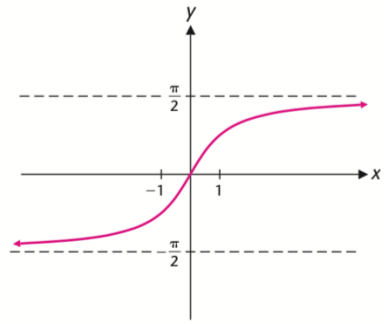
| | | | |
|---|---|---|---|
| $y = \sin^{-1} x$ | is equivalent to | $x = \sin y$ | where $-1 \le x \le 1$, $-\pi/2 \le y \le \pi/2$ |
| $y = \cos^{-1} x$ | is equivalent to | $x = \cos y$ | where $-1 \le x \le 1$, $0 \le y \le \pi$ |
| $y = \tan^{-1} x$ | is equivalent to | $x = \tan y$ | where $-\infty < x < \infty$, $-\pi/2 < y < \pi/2$ |



$y = \sin^{-1} x$
Domain = $[-1, 1]$
Range = $[-\frac{\pi}{2}, \frac{\pi}{2}]$



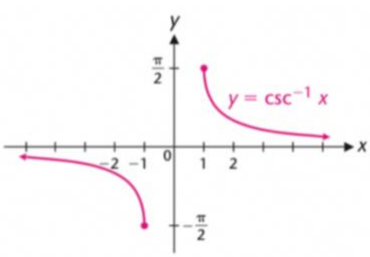
$y = \cos^{-1} x$
Domain = $[-1, 1]$
Range = $[0, \pi]$



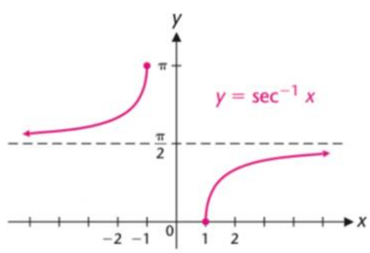
$y = \tan^{-1} x$
Domain = $(-\infty, \infty)$
Range = $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

## DEFINITION 4 Inverse Cotangent, Secant, and Cosecant Functions

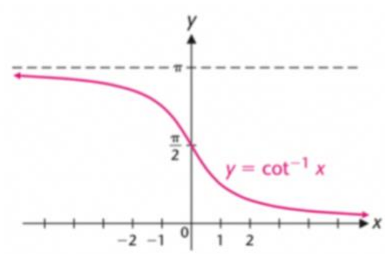
| | | | |
|---|---|---|---|
| $y = \csc^{-1} x$ | is equivalent to | $x = \csc y$ | where $-\pi/2 \le y \le \pi/2$, $y \ne 0$, $\lvert x\rvert \ge 1$ |
| $y = \sec^{-1} x$ | is equivalent to | $x = \sec y$ | where $0 \le y \le \pi$, $y \ne \pi/2$, $\lvert x\rvert \ge 1$ |
| $y = \cot^{-1} x$ | is equivalent to | $x = \cot y$ | where $0 < y < \pi$, $-\infty < x < \infty$ |




Domain: $x \le -1$ or $x \ge 1$
Range: $-\pi/2 \le y \le \pi/2$, $y \ne 0$




Domain: $x \le -1$ or $x \ge 1$
Range: $0 \le y \le \pi$, $y \ne \pi/2$




Domain: All real numbers
Range: $0 < y < \pi$

[*Note:* The domain restrictions used in defining $\sec^{-1}$ and $\csc^{-1}$ are not universally agreed upon.]