

**CS 231: Intro to Digital Circuits Practice Problems**

1. Draw the following logic gates and their truth tables respectively.

a. AND

b. OR

c. NOT

d. XOR

e. NOR

f. NAND

2. Complete using Boolean algebra:

a.  $A + 0 =$  \_\_\_\_\_

b.  $A + A' =$  \_\_\_\_\_

c.  $A + A =$  \_\_\_\_\_

d.  $A + 1 =$  \_\_\_\_\_

e.  $(A')' =$  \_\_\_\_\_

f.  $A \cdot 1 =$  \_\_\_\_\_

g.  $A \cdot A' =$  \_\_\_\_\_

h.  $A \cdot A =$  \_\_\_\_\_

i.  $A \cdot 0 =$  \_\_\_\_\_

3. Complete the following terms using each property of Boolean algebra:

a. Commutative:

$A + B =$  \_\_\_\_\_

$AB =$  \_\_\_\_\_

b. Associative:

$A + (B + C) =$  \_\_\_\_\_

$A(BC) =$  \_\_\_\_\_

c. Distributive:

$$A(B+C) = \underline{\hspace{2cm}}$$

$$A + BC = \underline{\hspace{2cm}}$$

d. DeMorgan:

$$(A + B)' = \underline{\hspace{2cm}}$$

$$(AB)' = \underline{\hspace{2cm}}$$

e. Absorption:

$$A + AB = \underline{\hspace{2cm}}$$

$$A(A + B) = \underline{\hspace{2cm}}$$

4. Find the compliment of the following functions by applying DeMorgan's theorem as necessary.

a.  $f = a'bc' + a'b'c$

b.  $f = a(b'c' + bc)$

c.  $f = AB + C'D' + B'D$

d.  $f = (ab' + c)d' + e$

e.  $F = x(y' + z)$

f.  $F = X(Y'Z + YZ)$

g.  $f = (a + b' + c)(a' + c')(a + b)$

h.  $f = ABCD + A'B'CD + ACD + A'B$

i.  $F = x'(yz) + w$

5. Given the following functions, find their representation in sum of minterms and product of maxterms format.

a.  $F = XY + X'Z$

b.  $f = (wx + y)(x + y'z)$

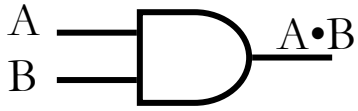
c.  $F = A'BC + A'B'C' + B'C$

d.  $F = a'b'c + a'b + ac + a'b'c'$

Solutions

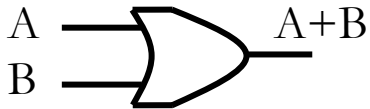
1. Draw the following logic gates and their truth tables respectively.

a. AND



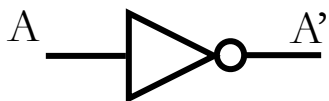
A	B	$A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

b. OR



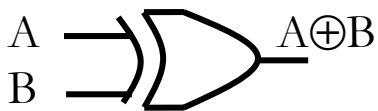
A	B	$A + B$
0	0	0
0	1	1
1	0	1
1	1	1

c. NOT



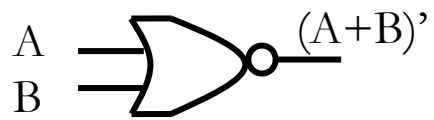
A	$A'$
0	1
1	0

d. XOR



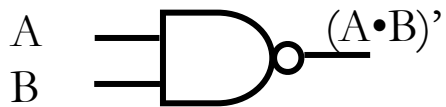
A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

e. NOR



A	B	$(A+B)'$
0	0	1
0	1	0
1	0	0
1	1	0

f. NAND



A	B	$(A \cdot B)'$
0	0	1
0	1	1
1	0	1
1	1	0

2. Complete using Boolean algebra:

a.  $A + 0 = A$

b.  $A + A' = 1$

c.  $A + A = A$

d.  $A + 1 = 1$

e.  $(A')' = A$

f.  $A \cdot 1 = A$

g.  $A \cdot A' = 0$

h.  $A \cdot A = A$

i.  $A \cdot 0 = 0$

3. Complete the following terms using each property of Boolean algebra:

a. Commutative:

$$A + B = B + A$$

$$AB = BA$$

b. Associative:

$$A + (B + C) = (A + B) + C$$

$$A (BC) = (AB) C$$

c. Distributive:

$$A (B+C) = AB + AC$$

$$A + BC = (A + B) (A + C)$$

d. DeMorgan:

$$(A + B)' = A'B'$$

$$(AB)' = A' + B'$$

e. Absorption:

$$A + AB = A$$

$$A (A + B) = A$$

4. Find the compliment of the following functions by applying DeMorgan's theorem as necessary.

a.  $f = a'bc' + a'b'c$

$$\begin{aligned} f' &= (a'bc' + a'b'c)' \\ &= (a'bc')' (a'b'c)' \\ &= (\mathbf{a + b' + c})(\mathbf{a + b + c'}) \end{aligned}$$

b.  $f = a(b'c' + bc)$



$$\begin{aligned}
 f' &= [a(b'c' + bc)]' \\
 &= a' + (b'c' + bc)' \\
 &= a' + (b'c')' (bc)' \\
 &= \mathbf{a' + (b + c)(b' + c')}
 \end{aligned}$$

c.  $f = AB + C'D' + B'D$

$$\begin{aligned}
 f' &= (AB + C'D' + B'D)' \\
 &= (AB)' (C'D')' (B'D)' \\
 &= \mathbf{(A' + B')(C + D)(B + D')}
 \end{aligned}$$

d.  $f = (ab' + c)d' + e$

$$\begin{aligned}
 f' &= [(ab' + c)d' + e]' \\
 &= ((ab' + c)d')' e' \\
 &= ((ab' + c)' + d) e' \\
 &= \mathbf{((a' + b)c + d) e'}
 \end{aligned}$$

e.  $F = x(y' + z)$

$$\begin{aligned}
 F' &= [x(y' + z)]' \\
 &= x' + (y' + z)' \\
 &= \mathbf{x' + yz'}
 \end{aligned}$$

f.  $F = X(Y'Z + YZ)$

$$\begin{aligned}
 F' &= [X(Y'Z + YZ)]' \\
 &= X' + (Y'Z + YZ)' \\
 &= \mathbf{X' + (Y + Z')(Y' + Z')}
 \end{aligned}$$

g.  $f = (a + b' + c)(a' + c')(a + b)$

$$\begin{aligned}
 f &= [(a + b' + c)(a' + c')(a + b)]' \\
 &= (a + b' + c)' + (a' + c')' + (a + b)' \\
 &= \mathbf{a'bc' + ac + a'b'}
 \end{aligned}$$

h.  $f = ABCD + A'B'CD + ACD + A'B$

$$\begin{aligned}
 f' &= [ABCD + A'B'CD + ACD + A'B]' \\
 &= (ABCD)' (A'B'CD)' (ACD)' (A'B)' \\
 &= \mathbf{(A' + B' + C' + D') (A + B + C' + D') (A' + C' + D') (A + B')}
 \end{aligned}$$

i.  $F = x^2(yz) + w$

$$\begin{aligned} F' &= [x^2(yz) + w]' \\ &= (x^2(yz))' w' \\ &= (x + (yz)') w' \\ &= (\mathbf{x + (y' + z')}) w' \end{aligned}$$

5. Given the following functions, find their representation in sum of minterms and product of maxterms format.

a.  $F = XY + X'Z$

X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Sum of minterms:

$$F = X'Y'Z + X'YZ + XYZ' + XYZ$$

Product of maxterms:

$$\begin{aligned} F' &= X'Y'Z' + X'YZ' + XY'Z' + XY'Z \\ (F') &= (X + Y + Z)(X + Y' + Z)(X' + Y + Z)(X' + Y' + Z) \end{aligned}$$

b.  $f = (wx + y)(x + y'z) = wx + wxy'z + yx$

W	X	Y	Z	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Sum of minterms:

$$f = w'xyz' + w'xyz + wxy'z' + wxy'z + wxyz' + wxyz$$

Product of maxterms:

$$f' = w'x'y'z' + w'x'y'z + w'x'yz' + w'x'yz + w'xy'z' + w'xy'z + wx'y'z' + wx'y'z + wx'yz' + wx'yz$$

$$(f')' = (w + x + y + z)(w + x + y + z')(w + x + y' + z)(w + x + y' + z')(w + x' + y + z)(w + x' + y + z')(w' + x + y + z)(w' + x + y + z')(w' + x + y' + z)(w' + x + y' + z')$$

c.  $F = A'BC + A'B'C' + B'C$

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Sum of minterms:

$$F = A'B'C' + A'B'C + A'BC + AB'C$$

Product of maxterms:

$$F' = A'BC' + AB'C' + ABC' + ABC$$

$$(F')' = (A + B' + C)(A' + B + C)(A' + B' + C)(A' + B' + C)$$

d.  $F = a'b'c + a'b + ac + a'b'c'$

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Sum of minterms:

$$F = a'b'c' + a'b'c + a'bc' + a'bc + ab'c + abc$$

Product of maxterms:

$$F' = ab'c' + abc'$$

$$(F')' = (a' + b + c)(a' + b' + c)$$