

Math 101 Chapter 4/Section: 3 Topic: Saving for the Long Term, Worksheet

Finish the formulas for the following in terms of:

t = number of deposits, y = years, r = monthly interest rate, APR = annual percentage rate

1. Balance after t monthly deposits =

2. Needed monthly deposit =

3. Monthly annuity yield =

4. Nest egg needed =

Match the following questions to the equations (1-4) above:

_____ How much money would you need to deposit each month in order to reach your desired result?

_____ If you start with money in the bank and withdraw the same amount of money for each month, how much would you be able to withdraw each month?

_____ How much money you would have after depositing money for t months?

_____ If you know how much money you want to withdraw each month for ' t ' months, how much do you have to start with

ANSWER KEY:

Finish the formulas for the following in terms of:

t = number of deposits, y = years, r = monthly interest rate, APR = annual percentage rate

*** Right Column:** This is when the rate (r) is given in the problem as a yearly rate (APR). The yearly rate (APR) must be divided into 12 months because the problem is asking about monthly deposits.

1. Balance after t monthly deposits =

$$\frac{\text{Deposit} \times \left((1+r)^t - 1 \right)}{r} \quad \text{OR} \quad \frac{\text{Deposit} \times \left(\left(1 + \frac{\text{APR}}{12} \right)^{(12y)} - 1 \right)}{\left(\frac{\text{APR}}{12} \right)}$$

2. Needed monthly deposit =

$$\frac{\text{Goal} \times r}{\left((1+r)^t - 1 \right)} \quad \text{OR} \quad \frac{\text{Goal} \times \left(\frac{\text{APR}}{12} \right)}{\left(\left(1 + \frac{\text{APR}}{12} \right)^{(12y)} - 1 \right)}$$

3. Monthly annuity yield =

$$\frac{\text{Nest egg} \times r \times (1+r)^t}{\left((1+r)^t - 1 \right)} \quad \text{OR} \quad \frac{\text{Nest egg} \times \frac{\text{APR}}{12} \times \left(1 + \frac{\text{APR}}{12} \right)^{(12y)}}{\left(\left(1 + \frac{\text{APR}}{12} \right)^{(12y)} - 1 \right)}$$

ANSWER KEY

4. Nest egg needed =

$$\frac{\text{Monthly annuity yield} \times ((1+r)^t - 1)}{(r \times (1+r)^t)} \quad \text{OR} \quad \frac{\text{Monthly annuity yield} \times \left(\left(1 + \frac{\text{APR}}{12} \right)^{(12y)} - 1 \right)}{\left(\frac{\text{APR}}{12} \times \left(1 + \frac{\text{APR}}{12} \right)^{(12y)} \right)}$$

Match the following questions to the equations (1-4) above:

__2__ How much money would you need to deposit each month in order to reach your desired result?

__3__ If you start with money in the bank and withdraw the same amount of money for each month, how much would you be able to withdraw each month?

__1__ How much money you would have after depositing money for t months?

__4__ If you know how much money you want to withdraw each month for 't' months, how much do you have to start with?

ANSWER KEY

Solve the following problems:

- You open a savings account and deposit \$200 into it at the end of each month. The account pays you a monthly interest rate of 1.5% on the balance in the account at the beginning of each month. At the end of the first month the balance is \$200. At the end of the second month the balance is \$403. Track the growth of this account through 6 months. (***Hint:** New balance = Previous balance + Interest + Deposit. You start with \$0 in your savings account.)

Use Formula:

$$\text{New balance} = \text{Previous balance} + \text{Interest} + \text{Deposit}$$

$$\text{Balance after 1 deposit} = \$0 + (\$0 \times 0.015) + \$200 = \$200$$

$$\text{Balance after 2 deposits} = \$200 + (\$200 \times 0.015) + \$200 = \$403$$

$$\text{Balance after 3 deposits} = \$403 + (\$403 \times 0.015) + \$200 = \$609.05$$

...and so on

At end of month #	Interest paid on previous balance	Deposit	New balance
1	\$0	\$200	\$200
2	1.5% of \$200	\$200	\$403
3	1.5% of \$403	\$200	\$609.05
4	1.5% of \$609.05	\$200	\$818.19
5	1.5% of \$818.18	\$200	\$1030.46
6	1.5% of \$1030.45	\$200	\$1245.92

ANSWER KEY

2. Suppose we have a savings account earning 8% APR. We deposit \$30 into the account at the end of each month for 4 years. What is the account balance after 4 years?

Use formula:

Balance after t monthly deposits =

$$\frac{\text{Deposit} \times \left(\left(1 + \frac{\text{APR}}{12} \right)^{(12y)} - 1 \right)}{\left(\frac{\text{APR}}{12} \right)}$$

$$= [\$30 \times ([1+0.08/12]^{(12 \times 4)} - 1)] / [0.08/12]$$

$$= 1690.497$$

The balance after 4 years is **\$1690.50**

3. How much do you need to deposit each month into your savings account that has an APR of 9% in order to have \$30,000 for your college education in 3 years?

Use formula:

Needed monthly deposit =

$$\frac{\text{Goal} \times \left(\frac{\text{APR}}{12} \right)}{\left(\left(1 + \frac{\text{APR}}{12} \right)^{(12y)} - 1 \right)}$$

$$= [\$30,000 \times (0.09/12)] / [(1+[0.09/12])^{(12 \times 3)} - 1]$$

$$= \$728.992$$

Round up because if you round down you will not reach your goal.

You need to deposit **\$729.00** monthly

ANSWER KEY

4. Suppose we have \$1,000,000 in the bank with an APR of 6.3% compounded monthly. Find the monthly yield for a 30-year annuity.

Use formula:

Monthly annuity yield =

$$\frac{\text{Nest egg} \times \frac{\text{APR}}{12} \times \left(1 + \frac{\text{APR}}{12}\right)^{(12y)}}{\left(\left(1 + \frac{\text{APR}}{12}\right)^{(12y)} - 1\right)}$$

$$= [\$1,000,000 \times (0.063/12) \times (1+0.063/12)^{(12 \times 30)}] / [(1+0.063/12)^{(12 \times 30)} - 1]$$

$$= \$6189.728$$

The monthly annuity yield is **\$6189.73**

5. Suppose your retirement account pays 4.9% APR compounded monthly. How much do you need in order to retire with a 20-year annuity that yields \$5,000 a month?

Use formula:

Nest egg needed =

$$\frac{\text{Annuity yield goal} \times \left(\left(1 + \frac{\text{APR}}{12}\right)^{(12y)} - 1\right)}{\left(\frac{\text{APR}}{12} \times \left(1 + \frac{\text{APR}}{12}\right)^{(12y)}\right)}$$

$$= [\$5,000 \times ((1+0.049/12)^{(12 \times 20)} - 1)] / [(0.049/12) \times (1+0.049/12)^{(12 \times 20)}]$$

$$= \$764,007.253$$

Round up because if you round down you will not reach your goal.

The nest egg needed is **\$764,007.26**

Citation: Crauder, Bruce. Quantitative Literacy.. [VitalSource Bookshelf].

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