

**Math 215 Chapter 7: Trigonometric Identities and Conditional Equations/Section Topics: 1-5**

**Verify Identity**

1.  $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$

2.  $\frac{\cos^2 x - \sin^2 x}{\sin x \cos x} = \cot x - \tan x$

3.  $\frac{3\cos^2 z + 5\sin z - 5}{\cos^2 z} = \frac{3\sin z - 2}{1 + \sin z}$

4.  $\tan\left(\frac{\pi}{2} - x\right) = \cot x$

5.  $\tan x - \tan y = \frac{\sin(x-y)}{\cos x \cos y}$

6.  $\cot 2x = \frac{\cot^2 x - 1}{2\cot x}$

7.  $\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}$

8.  $\frac{\cos x - \cos y}{\sin x + \sin y} = -\tan \frac{x-y}{2}$

**Evaluate Using Appropriate Identities**

9.  $\cos 105^\circ - \cos 15^\circ$

10.  $\sin 22^\circ \cos 38^\circ + \cos 22^\circ \sin 38^\circ$

11.  $\sec 75^\circ$

**Find exact solutions over the indicated intervals**

12.  $2\cos x - 1 = 0, 0 \leq x \leq 2\pi$

13.  $\sin^2 x + 2\cos x = -2, 0 \leq x \leq 2\pi$

14.  $2\sin^2 \frac{x}{2} - 3\sin \frac{x}{2} + 1 = 0, 0 \leq x \leq 2\pi$

15.  $\tan x - \sqrt{3} = 0, \text{ all real } x$

## Solutions

1.  $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$  Factor left side of equation
- $(\sin x + \cos x)(\sin x + \cos x) = 1 + 2\sin x \cos x$  Simplify
- $\sin^2 x + 2\sin x \cos x + \cos^2 x = 1 + 2\sin x \cos x$  Simplify
- $\sin^2 x + \cos^2 x + 2\sin x \cos x = 1 + 2\sin x \cos x$  Use pythagorean identity
- $1 + 2\sin x \cos x = 1 + 2\sin x \cos x$  Proven equal
- 
2.  $\frac{\cos^2 x - \sin^2 x}{\sin x \cos x} = \cot x - \tan x$  Factor left side of equation
- $\frac{(\cos x + \sin x)(\cos x - \sin x)}{\sin x \cos x} = \cot x - \tan x$
- $\frac{\cos x + \sin x}{\sin x} \times \frac{\cos x - \sin x}{\cos x} = \cot x - \tan x$  Separated the fraction using Multiplication
- $(\frac{\cos x}{\sin x} + \frac{\sin x}{\sin x}) \times (\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}) = \cot x - \tan x$  Separated the fractions using addition and subtraction
- $(\cot x + 1)(1 - \tan x) = \cot x - \tan x$  Simplified fractions
- $\cot x - \cot x \tan x + 1 - \tan x = \cot x - \tan x$  Foiled left side of equation
- $\cot x - 1 + 1 - \tan x = \cot x - \tan x$  Simplified  $\cot x \tan x = 1$
- $\cot x - \tan x = \cot x - \tan x$  Proven equal

3.  $\frac{3\cos^2 z + 5\sin z - 5}{\cos^2 z} = \frac{3\sin z - 2}{1 + \sin z}$       Use  $\cos^2 z = 1 - \sin^2 z$  to substitute left side

$\frac{3(1 - \sin^2 z) + 5\sin z - 5}{1 - \sin^2 z} = \frac{3\sin z - 2}{1 + \sin z}$       Simplify

$\frac{3 - 3\sin^2 z + 5\sin z - 5}{1 - \sin^2 z} = \frac{3\sin z - 2}{1 + \sin z}$       Simplify

$\frac{-3\sin^2 z + 5\sin z - 2}{1 - \sin^2 z} = \frac{3\sin z - 2}{1 + \sin z}$       Simplify

$\frac{-(3\sin z - 2)(\sin z - 1)}{(1 + \sin z)(1 - \sin z)} = \frac{3\sin z - 2}{1 + \sin z}$       Distribute -1 to  $(\sin z - 1)$

$\frac{(3\sin z - 2)(-\sin z + 1)}{(1 + \sin z)(1 - \sin z)} = \frac{3\sin z - 2}{1 + \sin z}$       Cancel out  $(-\sin z + 1)$  and  $(1 - \sin z)$

$\frac{3\sin z - 2}{(1 + \sin z)} = \frac{3\sin z - 2}{1 + \sin z}$       Proven equal

4.  $\tan\left(\frac{\pi}{2} - x\right) = \cot x$

$\frac{\sin\left(\frac{\pi}{2} - x\right)}{\cos\left(\frac{\pi}{2} - x\right)} = \cot x$       Convert tan to cos and sin on left side

$\frac{\sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x}{\cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x} = \cot x$       Used sin and cos difference identities

$\frac{\cos x}{\sin x} = \cot x$       Simplify

$\cot x = \cot x$       Proven equal

5.  $\tan x - \tan y = \frac{\sin(x-y)}{\cos x \cos y}$

$\frac{\sin x}{\cos x} - \frac{\sin y}{\cos y} = \frac{\sin(x-y)}{\cos x \cos y}$       Converted tan to sin and cos on left side

$\frac{\sin x \cos y}{\cos x \cos y} - \frac{\cos x \sin y}{\cos x \cos y} = \frac{\sin(x-y)}{\cos x \cos y}$       Found common denominator on left side

$$\frac{\sin x \cos y - \cos x \sin y}{\cos x \cos y} = \frac{\sin(x-y)}{\cos x \cos y}$$

Use difference identity on left side

$$\frac{\sin(x-y)}{\cos x \cos y} = \frac{\sin(x-y)}{\cos x \cos y}$$

Proven equal

6.  $\cot 2x = \frac{\cot^2 x - 1}{2 \cot x}$

$$\frac{\cos 2x}{\sin 2x} = \frac{\cot^2 x - 1}{2 \cot x}$$

Converted cot to sin and cos on left side

$$\frac{\cos^2 x - \sin^2 x}{2 \sin x \cos x} = \frac{\cot^2 x - 1}{2 \cot x}$$

Used double angle identities

$$\frac{\cos^2 x - \sin^2 x}{2 \sin x \cos x} = \frac{\frac{\cos^2 x - 1}{\sin^2 x}}{\frac{2 \cos x}{\sin x}}$$

Converted cot to sin and cos on right side

$$\frac{\cos^2 x - \sin^2 x}{2 \sin x \cos x} = \frac{\frac{\cos^2 x - \sin^2 x}{\sin^2 x}}{\frac{2 \cos x}{\sin x}}$$

Found common denominator on right side

$$\frac{\cos^2 x - \sin^2 x}{2 \sin x \cos x} = \frac{\frac{\cos^2 x - \sin^2 x}{\sin^2 x}}{\frac{2 \cos x}{\sin x}}$$

Simplify

$$\frac{\cos^2 x - \sin^2 x}{2 \sin x \cos x} = \frac{\cos^2 x - \sin^2 x}{\sin^2 x} \times \frac{\sin x}{2 \cos x}$$

Multiply by the reciprocal

$$\frac{\cos^2 x - \sin^2 x}{2 \sin x \cos x} = \frac{\cos^2 x - \sin^2 x}{2 \sin x \cos x}$$

Proven equal

7.  $\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}$

$$\left( \pm \sqrt{\frac{1 + \cos x}{2}} \right)^2 = \frac{1 + \cos x}{2}$$

Use half angle identity on left side

$$\frac{1 + \cos x}{2} = \frac{1 + \cos x}{2}$$

Proven equal

8.  $\frac{\cos x - \cos y}{\sin x + \sin y} = -\tan \frac{x-y}{2}$

$$\frac{-2\sin(\frac{x+y}{2})\sin(\frac{x-y}{2})}{2\cos(\frac{x+y}{2})\sin(\frac{x-y}{2})} = -\tan \frac{x-y}{2}$$

Use sum to product identity on left side

$$\frac{-\sin(\frac{x+y}{2})}{\cos(\frac{x+y}{2})} = -\tan \frac{x-y}{2}$$

Simplify

$$-\tan \frac{x-y}{2} = -\tan \frac{x-y}{2}$$

Proven equal

9.  $\cos 105^\circ - \cos 15^\circ$

$$\cos(150^\circ - 45^\circ) - \cos(60^\circ - 45^\circ)$$

$$\cos 150^\circ \cos 45^\circ + \sin 150^\circ \sin 45^\circ - (\cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ)$$

$$\frac{-\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} + \frac{1}{2} \times \frac{\sqrt{2}}{2} - \frac{1}{2} \times \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}$$

$$-\frac{\sqrt{6}}{2}$$

10.  $\sin 22^\circ \cos 38^\circ + \cos 22^\circ \sin 38^\circ$

Use Sum identity

$$\sin(22^\circ + 38^\circ)$$

$$\sin(60^\circ)$$

$$\frac{\sqrt{3}}{2}$$

11.  $\sec 75^\circ$

$$\frac{1}{\cos 75^\circ}$$

$$\frac{1}{\cos(45^\circ+30^\circ)}$$

$$\frac{1}{\cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ}$$

$$\frac{1}{\frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2}}$$

$$\frac{4}{\sqrt{6}-\sqrt{2}}$$

**12.**  $2\cos x - 1 = 0, 0 \leq x \leq 2\pi$

Isolate X

$$2\cos x = 1$$

$$\cos x = 1/2$$

Use unit circle

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

**13.**  $\sin^2 x + 2\cos x = -2, 0 \leq x \leq 2\pi$

$$(1 - \cos^2 x) + 2\cos x = -2$$

Isolate X

$$-\cos^2 x + 2\cos x + 1 = -2$$

$$-\cos^2 x + 2\cos x + 3 = 0$$

$$\cos^2 x - 2\cos x - 3 = 0$$

$$(\cos x + 1)(\cos x - 3) = 0$$

$$\cos x + 1 = 0, \cos x - 3 = 0$$

$$\cos x = -1, \cos x = 3$$

Use unit circle

$$x = \pi$$

14.  $2\sin^2\frac{x}{2} - 3\sin\frac{x}{2} + 1 = 0, 0 \leq x \leq 2\pi$

$$(2\sin\frac{x}{2} - 1)(\sin\frac{x}{2} - 1) = 0$$

Factored equation

$$2\sin\frac{x}{2} - 1 = 0, \sin\frac{x}{2} - 1 = 0$$

$$2\sin\frac{x}{2} = 1, \sin\frac{x}{2} = 1$$

$$\sin\frac{x}{2} = \frac{1}{2}, \sin\frac{x}{2} = 1$$

Use unit circle

$$x = \pi, x = \frac{\pi}{3} \& \frac{5\pi}{3}$$

15.  $\tan x - \sqrt{3} = 0, \text{ all real } x$

$$\tan x = \sqrt{3}$$

$$x = \frac{\pi}{3} + \pi n$$