

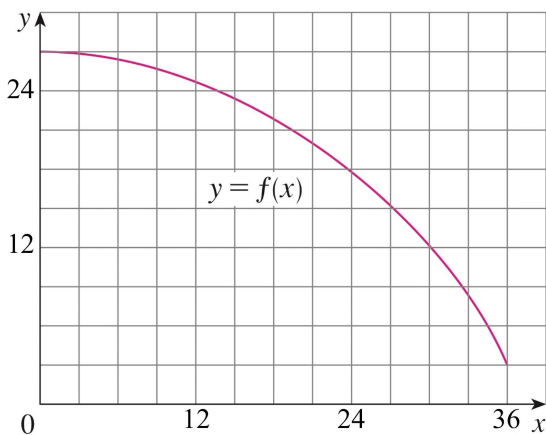
**Math 160 Chapter 5/ Section 1-5: Riemann Sum, Indefinite Integrals, and Definite integrals  
Worksheet**

**Write the equation of the following:**

1. Definition of the area under a curve:
  
  
  
  
  
  
  
  
  
  
  
2. Definition of a definite integral:
  
  
  
  
  
  
  
  
  
  
  
3. Midpoint Rule:

**Find the area under the curve:**

1. Use six rectangles to find estimates of each type for the area under the given graph of  $f$  from  $x = 0$  to  $x = 36$ .



- i.  $L_6$  (sample points are left endpoints). Is this an over or underestimate of the true area?
- ii.  $R_6$  (sample points are right endpoints). Is this an over or underestimate of the true area?

- iii.  $M_6$  (sample points are midpoints). Is this an over or underestimate of the true area?
2. Estimate the area under the curve of  $f(x) = x^2 - 1$  from  $x = 1$  to  $x = 5$  using 4 approximating rectangles and use left and right endpoints.
3. Estimate the area under the curve of  $f(x) = 4 \sin(x)$  from  $x = 0$  to  $x = \frac{3\pi}{2}$  using 6 approximating rectangles and use left and right endpoints.
4. Estimate the area under the curve of  $f(x) = \frac{1}{4}x^2 + 3$  from  $x = 0$  to  $x = 4$  using 4 approximating rectangles and use midpoints.

Evaluate the integrals:

1.  $\int (x^7 + x^4 + 2) dx$

2.  $\int \frac{r^5 + r^2 - r}{r^2} dr$

3.  $\int_1^3 (x^3 + 5x + 9) dx$

4.  $\int_{-2}^2 e^3 dx$

**Answer Sheet:**

Definitions:

1.  $A = \lim_{n \rightarrow \infty} L_n = \lim_{n \rightarrow \infty} [f(x_1) \Delta x + f(x_2) \Delta x + \dots + f(x_n) \Delta x]$
2.  $\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \left( \sum_{i=1}^n f(x_i) \Delta x \right)$ ,  $\Delta x = \frac{b-a}{n}$  and  $x_i$  is any point in the interval
3. Midpoint Rule:  $\int_a^b f(x) dx \approx \sum_{i=1}^n f(\bar{x}_i) \Delta x = \Delta x [f(\bar{x}_1) + \dots + f(\bar{x}_n)]$  where  
 $\Delta x = \frac{b-a}{n}$  and  $\bar{x}_i = \frac{1}{2}(x_{i-1}, x_i)$

Finding the area under the curve:

1.  $\Delta x = \frac{b-a}{2} = \frac{36-0}{6} = 6$   
 $f(x_1) = f(0) = 27$   
 $f(x_2) = f(6) \approx 26$   
 $f(x_3) = f(12) \approx 25$   
 $f(x_4) = f(18) \approx 22$   
 $f(x_5) = f(24) = 18$   
 $f(x_6) = f(30) = 12$   
 $f(x_7) = f(36) = 3$

Left endpoints:  $A = (27)(6) + (26)(6) + (25)(6) + (22)(6) + (18)(6) + (12)(6) = 780$

Right endpoints:

$A = (26)(6) + (25)(6) + (25)(6) + (22)(6) + (18)(6) + (12)(6) + (18)(6) = 636$

Midpoints:

$f(\bar{x}_1) = f(3) \approx 26.5$

$f(\bar{x}_2) = f(9) \approx 25.5$

$f(\bar{x}_3) = f(15) \approx 23$

$f(\bar{x}_4) = f(21) \approx 19.5$

$f(\bar{x}_5) = f(27) = 15$

$f(\bar{x}_6) = f(33) \approx 8.5$

$A = (26.5)(6) + (25.5)(6) + (23)(6) + (19.5)(6) + (15)(6) + (8.5)(6) = 708$

2.  $A = \sum_{i=1}^1 f(x_i) \Delta x$   
 $\Delta x = \frac{5-1}{4} = 1$

$$\begin{aligned}f(x_1) &= f(1) = (1)^2 - 1 = 0 \\f(x_2) &= f(2) = (2)^2 - 1 = 3 \\f(x_3) &= f(3) = (3)^2 - 1 = 8 \\f(x_4) &= f(4) = (4)^2 - 1 = 15 \\f(x_5) &= f(5) = (5)^2 - 1 = 24\end{aligned}$$

Left endpoints:  $A = (0)(1) + (3)(1) + (8)(1) + (15)(1) = 26$

Right endpoints:  $A = (3)(1) + (8)(1) + (15)(1) + (24)(1) = 50$

3. Left endpoint:  $\frac{2\pi + \pi\sqrt{2}}{2}$   
Right endpoint:  $\frac{\pi\sqrt{2}}{2}$

$$A = \sum_{i=1}^1 f(x_i) \Delta x$$

$$\Delta x = \frac{\frac{3\pi}{6}}{2} = \frac{\pi}{4}$$

$$f(x_1) = f(0) = 4\sin(0) = 0$$

$$f(x_2) = f\left(\frac{\pi}{4}\right) = 4\sin\left(\frac{\pi}{4}\right) = 2\sqrt{2}$$

$$f(x_3) = f\left(\frac{\pi}{2}\right) = 4\sin\left(\frac{\pi}{2}\right) = 4$$

$$f(x_4) = f\left(\frac{3\pi}{4}\right) = 4\sin\left(\frac{3\pi}{4}\right) = 2\sqrt{2}$$

$$f(x_5) = f(\pi) = 4\sin(\pi) = 0$$

$$f(x_6) = f\left(\frac{5\pi}{4}\right) = 4\sin\left(\frac{5\pi}{4}\right) = -2\sqrt{2}$$

$$f(x_7) = f\left(\frac{3\pi}{2}\right) = 4\sin\left(\frac{3\pi}{2}\right) = -4$$

Left endpoints:

$$A = (2\sqrt{2})(0) + (2\sqrt{2})\left(\frac{\pi}{4}\right) + (4)\left(\frac{\pi}{4}\right) + (2\sqrt{2})\left(\frac{\pi}{4}\right) + (0)\left(\frac{\pi}{4}\right) + (-2\sqrt{2})\left(\frac{\pi}{4}\right) = \frac{2\pi + \pi\sqrt{2}}{2}$$

Right endpoints:

$$A = (2\sqrt{2})\left(\frac{\pi}{4}\right) + (4)\left(\frac{\pi}{4}\right) + (2\sqrt{2})\left(\frac{\pi}{4}\right) + (0)\left(\frac{\pi}{4}\right) + (-2\sqrt{2})\left(\frac{\pi}{4}\right) + (-4)\left(\frac{\pi}{4}\right) = \frac{\pi\sqrt{2}}{2}$$

4.  $\frac{69}{4}$

$$f(0.5) = \frac{1}{4}(0.5)^2 + 3 = \frac{49}{16}$$

$$f(1.5) = \frac{1}{4}(1.5)^2 + 3 = \frac{57}{16}$$

$$f(2.5) = \frac{1}{4}(2.5)^2 + 3 = \frac{73}{16}$$

$$f(3.5) = \frac{1}{4}(3.5)^2 + 3 = \frac{97}{16}$$

$$A = \left(\frac{49}{16}\right)(1) + \left(\frac{57}{16}\right)(1) + \left(\frac{73}{16}\right)(1) + \left(\frac{97}{16}\right)(1) = \frac{276}{16} = \frac{69}{4}$$

Evaluate the integrals:

1.  $\frac{x^8}{8} + \frac{x^5}{5} + 2x + C$

Solution:  $\frac{x^{7+1}}{7+1} + \frac{x^{4+1}}{4+1} + \frac{2x^{0+1}}{0+1} + C$   
 $\frac{x^8}{8} + \frac{x^5}{5} + 2x + C$

2.  $\frac{r^4}{4} + r + \ln(r) + C$

Solution:  $\int \frac{r^5+r^2+r}{r^2} = \int \frac{r^5}{r^2} + \frac{r^2}{r^2} + \frac{r}{r^2} = \int r^3 + 1 + \frac{1}{r}$   
 $\frac{r^{3+1}}{3+1} + \frac{r^{0+1}}{0+1} + \ln(r) + C$ , the antiderivative of  $\frac{1}{r} = \ln(r)$

3. 232

Solution:  $\frac{x^{3+1}}{3+1} + \frac{5x^{1+1}}{1+1} + \frac{9x^{0+1}}{0+1} = \frac{x^4}{4} + \frac{5x^2}{2} + 9x \Big|_1^3$   
 $\left[\frac{(3)^4}{4} + \frac{5(3)^2}{2} + 9(3)\right] - \left[\frac{(1)^4}{4} + \frac{5(1)^2}{2} + 9(1)\right]$   
 $= 232$

4.  $4e^3$

Solution:  $\int e^3 dx = e^3 x \Big|_{-2}^2$ , remember  $e^3$  is just a constant

$$[e^3(2)] - [e^3(-2)] = 2e^3 + 2e^3$$

$$= 4e^3$$